SIMULATOR AERO MODEL IMPLEMENTATION

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SUMMARY

A general discussion of the type of mathematical model used in a real-time, flight simulation is presented. It is recommended that the approach to math model development include modularity and standardization as modification and maintenance of the model will be much more efficient with this approach. The general equations of motion for an aircraft are developed in a form best suited to real time simulation. Models for a few helicopter subsystems are discussed in terms of general approaches that are commonly taken in today's simulations.

INTRODUCTION

This chapter is intended to provide the reader with a understanding of the type of mathematical model used in a real-time flight simulation. A flight simulation system is studied in order to gain a better understanding of the real (or planned) aircraft. The understanding or knowledge to be gained can be in a wide variety of areas including, engineering studies, handling qualities, or pilot training. To properly perform simulation, we need a total system, including a mathematical model, simulator hardware, visual system, and motion system, whose behavior is sufficiently similar to the real aircraft in the areas under investigation. The mathematical model is a key part of this total simulation system.

Before delving into some of the technical aspects, it is important to examine what is meant by the phrase "mathematical model", or more simply, "math model". Is it the equations and data that describe the aircraft's behavior? Is it the computer program and all the associated data used to compute where the aircraft is and what it is doing? Does it include "fudge factors" used to "tune-up" the simulation's performance? What about all the integration algorithms and numerical "tricks" to improve the solution; are they part of the math model? Naturally, there are different opinions as to what the math model includes; the following provides a definition useful to the present discussion.

The mathematical model is the description and specification of the aircraft's dynamic behavior. This "behavior" comprises the various motions of the airframe and the states and performance of the various subsystems, such as the engine, landing gear, and avionics. The math model considers all the external and internal influences on the aircraft and defines the resultant states of the aircraft and it's subsystems.

The model is best presented by mathematical equations, schematic diagrams, logic diagrams, tables and graphs of data including geometry, aerodynamic coefficients, gain schedules, etc. These engineering equations, diagrams, and data comprise the mathematical model.

The digital computer program and its associated data is an implementation of the mathematical model. It is not the model itself, but rather a method (non-unique) of computing the model. The answers obtained via these calculations are used to validate the mathematical model. The basic idea of validation (covered elsewhere in this volume)

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is twofold. One issue is to verify whether or not the program accurately solves the equations of the model. The other issue is to validate that the model accurately describes (predicts) the behavior of the real aircraft.

SCOPE

This discussion of simulation mathematical modeling will describe the general elements of a model. The complete model can be thought of in two parts, the general equations of motion and the specific aircraft subsystems. The general equations of motion for a rigid body will be derived. Since these equations and their derivation are well known, not all details will be given here. Emphasis will be given to a particular form of the general equations that is best suited to real time simulation. Modeling methods for various subsystems such as aerodynamics, engine, control systems, and so forth will be discussed in much less detail, without derivation or specific model examples. The subsystems are, of course, specific to a given aircraft design and configuration. Some general approaches used in today's simulations will be discussed.

THE SIMULATION MATHEMATICAL MODEL

Model Structure

The common and highly recommended approach to math model development is to employ the principles of modularity and standardization. Later, when the computer program implementation of the math model is designed, these principles will provide a significant pay-off in efficiency and maintainability. A modular approach requires that the model be broken down into smaller parts that share information and interact in clear and meaningful ways. The modules usually follow along lines that correspond to the physical components of the aircraft (the program design may not). Standardization of axis systems, nomenclature, and various mathematical and engineering conventions also has significant benefits both in the programming implementation and in communicating the meaning of the model to others. These principles have great economic benefit if more than one type or model of aircraft is to be simulated. Modules can be updated or replaced without a complete program rewrite. Also, standardization allows modules to be shared between different simulations.

With a modular approach to math modeling, it becomes necessary to define information interfaces. What does each module need for input? What does it provide as output to other modules? In some cases, an output requirement is defined by some other module. For instance, the rotor module may not actually need to compute "horsepower required" to model all the rotor states, but a transmission module may require it. Some modules will require inputs and/or will provide outputs to the "outside world", that is, external to any modeling. Pilot controls are an example of an external input and pilot station accelerations might be an example of a output needed for a motion generation system, but not by the model itself.

In Fig. 1 below, a typical model organization is depicted. The bubbles indicate a particular module and the lines and arrows indicate information flow. Certainly there will be numerous other modules and other information paths as dictated by the actual aircraft's subsystems. Also, it would be common for nearly all the modules to have access to the environmental parameters and the aircraft's state variables.

The modular breakdown of the math model shown in Fig. 1 suggests a major design approach and some guidelines for standardization. The modules to the left of the summation bubble all are specific to a particular aircraft. The summation, equations of motion, and environment are general to all aircraft and can form the standard or "core" modules for any simulation. The specific description of a particular aircraft should include the aerodynamic, propulsion, and landing gear forces and moments acting on the aircraft center of gravity. The standard modules sum the reactions and produce aircraft state variables (as well as numerous ancillary computations).

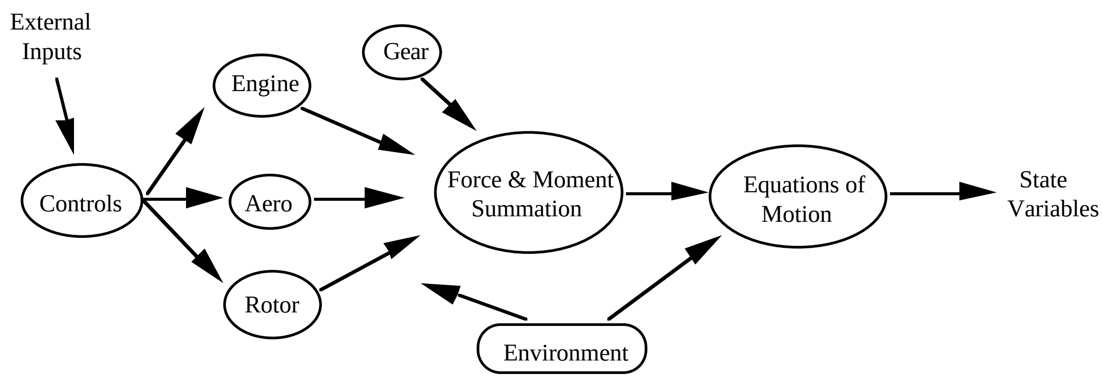

Figure 1 Typical Model Organization

The math model will be examined in two steps. First the general equations of motion that form the core of the simulation will be developed. All the equations necessary to predict the aircraft's state when the external forces and moments are known will be derived. A standard text on dynamics should provide any details omitted in this treatment (Refs. 1, 2). Next, some of the typical aircraft specific modules will be examined. The equations for these modules will not be derived or presented in any detail. Instead, a general discussion will give the reader some idea of what might be included in an actual model.

General Equations of Motion

An aircraft in flight has six degrees of freedom, three translational and three rotational. If the equations are written relative to the aircraft's center of mass, the translational and rotational sets of equations are independent of each other and can be derived separately. For this derivation, a non-rotating earth will be assumed which simplifies the understanding of the equations. If required, the effects of the earth's rotation can be added to the general equations at a later time without much difficulty. Also, wind and turbulence are not considered in the derivation that follows. Guidance on how air disturbances may be added is given in a later section.

Axis Systems. Before beginning the derivation, the various axis systems that are commonly used in flight simulation will be discussed. These axis systems are germane to the equations of motion. Other axis systems will be mentioned when discussing some of the aircraft subsystems.

<u>Body Axis.</u> The equations of motion are written with respect to the body axis system. Referring to Fig. 2, the body axis has its origin at the aircraft center of gravity, CG. The

x-axis is pointed forward out the nose; the y-axis is pointed out the right side of the aircraft, and the z-axis is directed down through the underside of the aircraft. The body axis is fixed to the aircraft and moves along with it. It forms a right-handed triad.

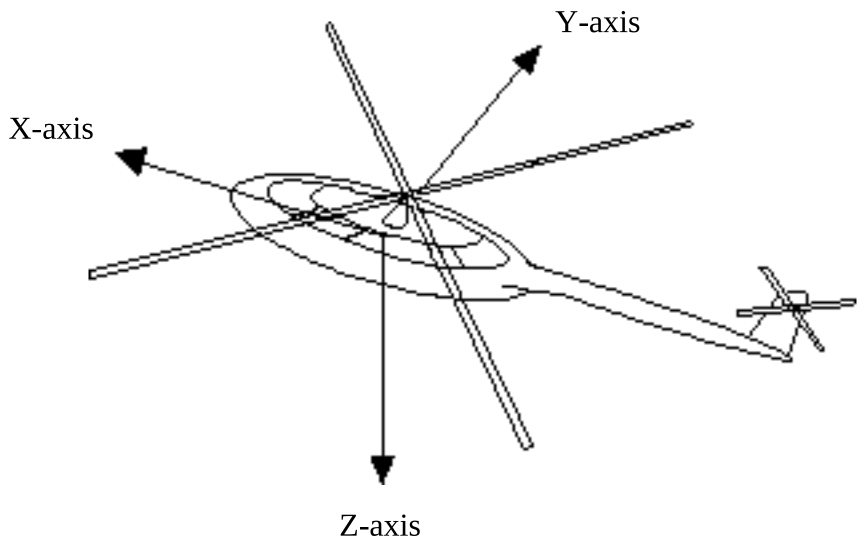

Figure 2 Body Axis System

Local Axis System. The local axis system also has its origin at the aircraft's CG, but has a fixed orientation. The X_L-axis always points north, the Y_L-axis points east, and the Z_L-axis points down. The local frame is depicted in Fig. 3.

Inertial Axis System. For purposes of this discussion, an inertial frame is taken as one that has its origin fixed at some point on the earth's surface. (For a rotating earth, placing the origin at the earth's center would be a better choice.) Its fixed orientation is the same as the Local frame and is also depicted in Fig. 3.

Turning attention once again to the body axis system, some other definitions and relationships can be developed. Whenever the x, y, or z axes are referred to without any subscripts, the body axis is implied. The six body axis velocities are shown in Fig. 3. The terms u, v, and w are the body axis translational velocities in the x, y, and z directions, respectively. The terms p, q, and r are the body axis rotational velocities about the x, y, and z axes, respectively.

The aircraft orientation is described by an ordered set of three Euler angles, ψ, θ, and ϕ, that relate the orientation of the body axis relative to the local axis system. In Fig. 4, the axis system designated by X_1, Y_1, and Z_1, is the initial reference orientation (in this work, it corresponds to the Local frame). First, the aircraft is given a rotation, ψ, about the Z_1 axis in the sense shown in the figure. This aligns the body axis with the system labeled X_2, Y_2, and Z_2. Next, the aircraft is rotated by, θ, about the Y_2 axis. The result is now the X_3, Y_3, and Z_3 frame. Finally, the aircraft is rotated by, ϕ, about the X_3 axis, yielding the X, Y, Z frame, the final orientation of the body axis system.

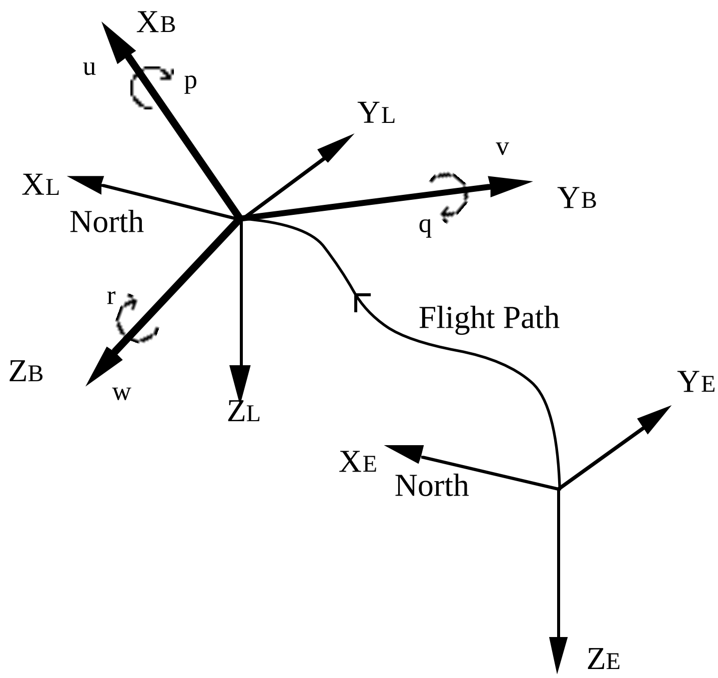

Figure 3 Inertial (E), Local (L), And Body (B) Axes

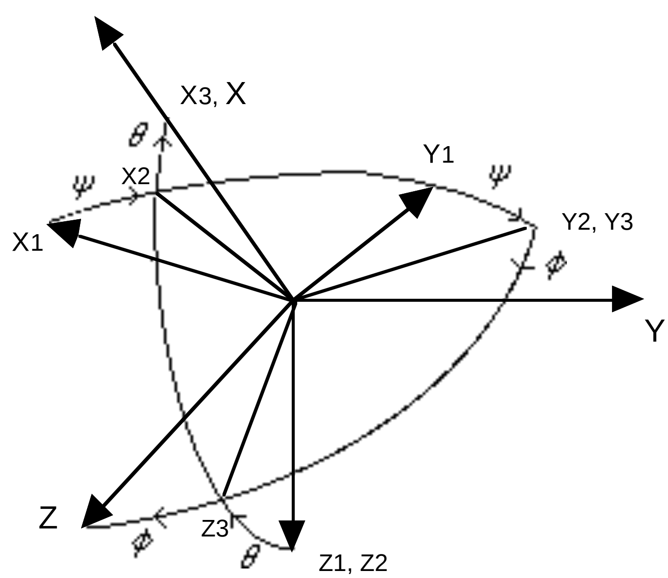

Figure 4 Aircraft Orientation Using Euler Angles

A pair of useful transformation matrices are those that rotate a vector from the Local to Body axis system and from the Body to Local axis system. Referring again to Fig. 4, the following three equations can be written.

The first rotation, ψ, results in

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Next, the rotation, θ, results in

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}$$

And, finally, the rotation ϕ, yields

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix}$$

Combining the three matrix multiplications into one, results in

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z1 \end{bmatrix}$$

This transformation matrix will be referred to as $[LtoB]$ (read "local to body"), and the above equation can be written as

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = [LtoB] \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix}$$

Also, since the transformation matrix is orthogonal, the inverse matrix can be obtained by a simple transpose. Therefore,

$$[BtoL] = [LtoB]^{-1} = [LtoB]^T$$

and

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = [BtoL] \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$

Translational Equations. The equations of motion are developed in the body axis system since the external forces are most easily defined in a coordinate system fixed in the aircraft. If $\vec{F}$ is the total external force acting on the aircraft, $\vec{v}$ is the absolute velocity of the center of mass, and m is the aircraft mass, then the following can be written.

$$\vec{F} = m\dot{\vec{v}}$$

The $\vec{F}$ and $\vec{v}$ vectors may be written in terms of their x, y, and z components.

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$
$$\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$$

where $\hat{i}, \hat{j}$, and $\hat{k}$ are an orthogonal triad of unit vectors aligned with the x, y, and z body axes, respectively. If the aircraft (and the body axis frame) is rotating with angular velocity $\vec{\omega}$, then the absolute acceleration can be found as follows.

$$\dot{\vec{v}} = (\dot{\vec{v}})_r + \vec{\omega} \times \vec{v}$$

Where $(\dot{\vec{v}})_r$ is the acceleration as viewed from the body axis system and the "$\times$" symbol signifies a vector cross product. This expression for the rate of change of a vector in a rotating system will be used repeatedly in the following derivations. The relative acceleration can be expanded as:

$$(\dot{\vec{v}})_r = \dot{u}\hat{i} + \dot{v}\hat{j} + \dot{w}\hat{k}$$

Also, the rotational velocity vector can be written as:

$$\vec{\omega} = p\hat{i} + q\hat{j} + r\hat{k}$$

Now the cross product can be evaluated and written as follows.

$$\vec{\omega} \times \vec{v} = (wq - vr)\hat{i} + (ur - wp)\hat{j} + (vp - uq)\hat{k}$$

Gathering terms and writing the vector equation in terms of three scalar relationships, we have:

$$F_x = m(\dot{u} + wq - vr)$$
$$F_y = m(\dot{v} + ur - wp)$$
$$F_z = m(\dot{w} + vp - uq)$$

Note that in some formulations, the gravitational force terms are explicitly written at this juncture. They are not included here for reasons to be made clear later. However, for illustration, the three force components could be written as follows.

$$F_x = F_{x\,Applied} - mg\sin\theta$$
$$F_y = F_{y\,Applied} + mg\cos\theta\sin\phi$$
$$F_z = F_{z\,Applied} + mg\cos\theta\cos\phi$$

Rotational Equations. If $\vec{M}$ is the total external moment acting at the center of mass and $\vec{H}$ is the angular momentum, then the following can be stated.

$$\vec{M} = \dot{\vec{H}}$$

The applied moment, $\vec{M}$, can be written as:

$$\vec{M} = L\hat{i} + M\hat{j} + N\hat{k}$$

The first task will be to write an expression for the angular momentum, then find its rate of change with respect to time. The angular momentum of a rigid body about it center of mass is given by

$$\vec{H} = \sum_i \vec{R}_i \times m_i \dot{\vec{R}}_i$$

where $\vec{R}$ is the position vector of a particle m_i. Assuming the position of each particle is fixed in the body, then the velocity of each particle is simply

$$\dot{\vec{R}}_i = \vec{\omega} \times \vec{R}_i$$

where $\vec{\omega}$ is the absolute angular velocity of the body. Further assuming that the body has a continuous mass distribution, the particle mass can be represented by its density times an elemental volume, ρdV (mass density is ρ). Substituting the expressions for velocity and elemental volume, the summation can be replaced by an integral as follows.

$$\vec{H} = \int_V \rho\vec{R} \times (\vec{\omega} \times \vec{R}) dV$$

Let $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ for a given elemental volume and, as previously, $\vec{\omega} = p\hat{i} + q\hat{j} + r\hat{k}$. The vector cross products can be written as follows.

$$\begin{aligned}\vec{R} \times (\vec{\omega} \times \vec{R}) = &\left[(y^2 + z^2)p - xyq - xzr\right]\hat{i} \\ &+ \left[-yxp + (x^2 + z^2)q - yzr\right]\hat{j} \\ &+ \left[-zxp - zyq + (x^2 + y^2)r\right]\hat{k}\end{aligned}$$

Now, the moments and products of inertia can be defined as

$$I_{xx} = \int_V \rho(y^2 + z^2)dV$$
$$I_{yy} = \int_V \rho(x^2 + z^2)dV$$
$$I_{zz} = \int_V \rho(x^2 + y^2)dV$$
$$I_{xy} = I_{yx} = \int_V \rho xydV$$
$$I_{xz} = I_{zx} = \int_V \rho xzdV$$
$$I_{yz} = I_{zy} = \int_V \rho yzdV$$

Substituting these definitions, plus the cross product terms, the angular momentum becomes

$$\begin{aligned}\vec{H} &= (I_{xx}p - I_{xy}q - I_{xz}r)\hat{i} \\ &\quad + (-I_{yx}p + I_{yy}q - I_{yz}r)\hat{j} \\ &\quad + (-I_{zx}p - I_{zy}q + I_{zz}r)\hat{k}\end{aligned}$$

For the typical aircraft case of symmetry with respect to the xz-plane

$$I_{xy} = I_{yx} = I_{yz} = I_{zy} = 0$$

Simplifying the expression for angular momentum, we have

$$\vec{H} = (I_{xx}p - I_{xz}r)\hat{i} + (I_{yy}q)\hat{j} + (-I_{xz}p + I_{zz}r)\hat{k}$$

Now, turning attention back to the rotational equation of motion, the time rate of change of angular momentum can be written as

$$\begin{aligned}\vec{M} &= \dot{\vec{H}} \\ &= (\dot{\vec{H}})_r + \vec{\omega} \times \vec{H}\end{aligned}$$

where $(\dot{\vec{H}})_r$ is the rate of change of angular momentum observed from the body axis system (rotating system). Expanding terms

$$(\dot{\vec{H}})_r = (I_{xx}\dot{p} - I_{xz}\dot{r})\hat{i} + I_{yy}\dot{q}\,\hat{j} + (-I_{xz}\dot{p} + I_{zz}\dot{r})\hat{k}$$

and,

$$\bar{\omega} \times \bar{H} = (H_z q - H_y r)\hat{i} + (H_x r - H_z p)\hat{j} + (H_y p - H_x q)\hat{k}$$

$$\begin{aligned} &= \left[(-I_{xz} p + I_{zz} r)q - (I_{yy} q)r\right]\hat{i} \\ &\quad + \left[(I_{xx} p - I_{xz} r)r - (-I_{xz} p + I_{zz} r)p\right]\hat{j} \\ &\quad + \left[(I_{yy} q)p - (I_{xx} p - I_{xz} r)q\right]\hat{k} \end{aligned}$$

Collecting terms and writing in terms of three scalar equations.

$$\begin{aligned} L &= I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xz} pq + (I_{zz} - I_{yy})qr \\ M &= I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2) \\ N &= I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{yy} - I_{xx})pq + I_{xz} rq \end{aligned}$$

The above three equations, plus the three translational equations comprise the equations of motion for the rigid body aircraft. However, a number of problems arise if these equations are used to compute the aircraft's dynamic state for simulation purposes.

With the translational equations, one problem is that the body axis is moving along the flight path and so the position of the aircraft relative to some fixed point (like a runway) is not easily specified. Another problem or "undesirable" feature is the presence of the various products involving rotational velocity terms, such as (wq) and (vr). These terms could cause problems with numerical accuracy when rotational velocities have large magnitude or frequency content. This situation is remedied by a transformation to an inertial frame, located on the earth's surface.

For the rotational equations, one problem is that the equations are coupled via $\dot{p}$, $\dot{q}$, and $\dot{r}$, which can lead to complications during numerical integration. To resolve that problem, the three equations can easily be decoupled by algebraic manipulation. Another issue is that the aircraft orientation cannot be found by simply integrating the rotational velocities. A change of variables is required to some other set of parameters, such as Euler angles.

Transformation of Translational Equations to an Inertial Frame. For the flat, non-rotating earth considered here, any fixed frame of reference can be employed as an inertial frame. The three forces acting on the aircraft center of gravity in the body axis system are rotated back through the Euler angles to the local frame and translated back to some convenient origin.

Again, the rotation from body axes to the local frame is given by

$$\begin{bmatrix} F_N \\ F_E \\ F_D \end{bmatrix} = [BtoL] \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

Where the subscripts, N, E, and D designate north, east, and down pointing directions. The accelerations are then

$$\begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = \begin{bmatrix} F_N / m \\ F_E / m \\ F_D / m \end{bmatrix}$$

A note can be made here about computer program implementation. If the body axis forces are taken so as to not include the gravitational terms, then the aircraft's weight can be added into the above equation as follows.

$$\begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = \begin{bmatrix} F_N / m \\ F_E / m \\ (F_D + weight) / m \end{bmatrix}$$

This implementation avoids having to rotate the gravitational terms twice. That is, instead of projecting the aircraft weight into the body axis and then rotating the summed forces into the inertial frame, the above suggestion allows the weight to be summed directly into the "down" force component in the inertial frame.

The accelerations can then be integrated to get velocities in the local frame and integrated again for displacements. The displacements can be referenced to any convenient location such as a runway threshold, navigation waypoint, or whatever is most useful to a simulation. The set of translational equations previously defined in the body axis system will prove useful for computing body axis accelerations at a later time.

Modifications to the Rotational Equations. The equations for L, M, and N are manipulated to yield $\dot{p}$, $\dot{q}$, and $\dot{r}$ on the left-hand side. The results are the following.

$$\begin{aligned}
\dot{p} &= \frac{I_{zz}}{D} L + \frac{I_{xz}}{D} N + \left[\frac{(I_{xx} - I_{yy} + I_{zz}) I_{xz}}{D} \right] pq + \left[\frac{(I_{yy} - I_{zz}) I_{zz} - I_{xz}{}^2}{D} \right] rq \\
\dot{q} &= \frac{1}{I_{yy}} M + \frac{(I_{zz} - I_{xx})}{I_{yy}} pr + \frac{I_{xz}}{I_{yy}} (r^2 - p^2) \\
\dot{r} &= \frac{I_{xz}}{D} L + \frac{I_{xx}}{D} N + \left[\frac{(I_{xx} - I_{yy}) I_{xx} + I_{xz}{}^2}{D} \right] pq + \left[\frac{(I_{yy} - I_{xx} - I_{zz}) I_{xz}}{D} \right] rq
\end{aligned}$$

where D is defined as

$$D = I_{XX} I_{ZZ} - I_{XZ}^2$$

These uncoupled equations can now be integrated with respect to time to obtain p, q, and r. The resultant rotational velocities cannot be integrated to get angular displacements. That is, one cannot find a set of three parameters which define the aircraft's orientation and have p, q, and r as their time derivatives. In some formulations, direction cosines are used to describe the angular orientation. This involves nine direction cosines and six equations of constraint. One can also use various four parameter systems, so called quaternions. Quaternions have the advantage of not having singularities (gimbal lock), having one half the frequency content, and involve one equation of constraint. Aircraft orientation can also be specified by the use of Euler angles. Remember that Euler angles are an ordered set of three parameters and that there will be a singularity when the aircraft is pointed straight up or down.

What is needed is a relationship between time rates of change of the Euler angles and the body axis rotational rates. Assume that $\dot{\vec{\phi}}$, $\dot{\vec{\theta}}$, and $\dot{\vec{\psi}}$ are the angular velocity vectors associated with rates of change in the corresponding Euler angles. Then the total rotational rate vector can be written as

$$\vec{\omega} = \dot{\vec{\psi}} + \dot{\vec{\theta}} + \dot{\vec{\phi}}$$

Note that $\dot{\vec{\phi}}$, $\dot{\vec{\theta}}$, and $\dot{\vec{\psi}}$ are not a mutually orthogonal vector triad, but are non-orthogonal components of $\vec{\omega}$. Referring to Fig. 5,

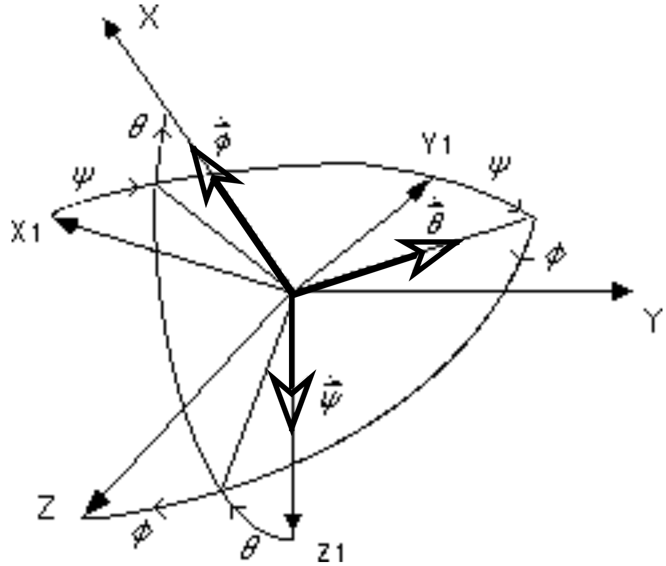

Figure 5 Euler Angle Rates

relationships can be written between the body axis rotational rates and $\dot{\vec{\phi}}$, $\dot{\vec{\theta}}$, and $\dot{\vec{\psi}}$ by summing the orthogonal projections of $\dot{\vec{\phi}}$, $\dot{\vec{\theta}}$, and $\dot{\vec{\psi}}$ onto the x, y, and z axes.

$$p = \dot{\phi} - \dot{\psi}\sin\theta$$
$$q = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi$$
$$r = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$$

or conversely,

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta$$
$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$$

Now with this last transformation we can change from the body axis rotational rates to Euler angle rates. The Euler angle rates can be integrated with respect to time to yield the Euler angles, thus specifying the aircraft's orientation. Keep in mind that the relationships are undefined at $\theta = \pm\frac{\pi}{2}$. Though not presented here, there are methods of treating this singularity if required.

Other Relationships. There are a number of other important relationships between the various state variables. Once the translational accelerations in the local frame are formed, they can be integrated to obtain velocities and again for displacements, thus

$$\begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = \int \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} dt$$

and

$$\begin{bmatrix} D_N \\ D_E \\ D_D \end{bmatrix} = \int \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} dt$$

The velocities can be transformed to the body axis system to yield u, v, and w.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = [LtoB] \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

Using the translational equations of motion that were written in the body axis system, the body axis accelerations can be solved for as follows.

$$\begin{aligned} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} &= \begin{bmatrix} vr - wq \\ wp - ur \\ uq - vp \end{bmatrix} + \begin{bmatrix} F_x / m \\ F_y / m \\ F_z / m \end{bmatrix} \\ &= \begin{bmatrix} vr - wq \\ wp - ur \\ uq - vp \end{bmatrix} + [LtoB] \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} \end{aligned}$$

The body axis velocities can be used to obtain the aircraft angle of attack and sideslip.

$$\alpha = \tan^{-1}(\frac{w}{u})$$

$$\beta = \tan^{-1}(\frac{v}{\sqrt{(u^2 + w^2)}})$$

Equation Summary. Given that the forces and moments acting on the aircraft's CG have been summed, the translation dynamics are described by the following equations.

$$\begin{bmatrix} F_N \\ F_E \\ F_D \end{bmatrix} = [BtoL] \begin{bmatrix} F_x' \\ F_y' \\ F_z' \end{bmatrix}$$

(where the primes indicates body axis forces without gravitational terms)

$$\begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = \begin{bmatrix} F_N / m \\ F_E / m \\ (F_D + weight) / m \end{bmatrix}$$

$$\begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = \int \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} dt$$

$$\begin{bmatrix} D_N \\ D_E \\ D_D \end{bmatrix} = \int \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} dt$$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} vr - wq \\ wp - ur \\ uq - vp \end{bmatrix} + [LtoB] \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = [LtoB] \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$$

The rotational dynamics are defined by the following equations.

$$\dot{p} = \frac{I_{zz}}{D} L + \frac{I_{xz}}{D} N + \left[\frac{(I_{xx} - I_{yy} + I_{zz}) I_{xz}}{D} \right] pq + \left[\frac{(I_{yy} - I_{zz}) I_{zz} - I_{xz}{}^2}{D} \right] rq$$

$$\dot{q} = \frac{1}{I_{yy}} M + \frac{(I_{zz} - I_{xx})}{I_{yy}} pr + \frac{I_{xz}}{I_{yy}} (r^2 - p^2)$$

$$\dot{r} = \frac{I_{xz}}{D} L + \frac{I_{xx}}{D} N + \left[\frac{(I_{xx} - I_{yy}) I_{xx} + I_{xz}{}^2}{D} \right] pq + \left[\frac{(I_{yy} - I_{xx} - I_{zz}) I_{xz}}{D} \right] rq$$

$$D = I_{XX} I_{ZZ} - I_{XZ}^2$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \int \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} dt$$

$$\dot{\psi} = (q \sin\phi + r \cos\phi) \sec\theta$$

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

$$\dot{\phi} = p + (q \sin\phi + r \cos\phi) \tan\theta$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \int \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} dt$$

Thus the aircraft's acceleration, velocity, location, and orientation are completely specified.

Aircraft Subsystem Modules

As was shown in Fig. 1, the various aircraft subsystems, such as aero, landing gear, and engine, feed information into the general equations of motion. These subsystems are what distinguish a particular aircraft simulation from another and as such cannot really be generalized to any degree. A few of the subsystems will be discussed in terms of general approaches that are commonly taken in today's simulations.

The control system is an example of a subsystem for which it is difficult to generalize an approach. It usually has a primary, or direct pilot input path and an augmenting or stabilizing path. The control laws may be implemented by either digital or analog circuits or both. The model might include a representation of actuator dynamics. In any case, the control system should be specified by system diagrams, logic statements, data values, gain schedules, etc. In the computer program implementation of the control system model there is a need to have a good transfer function solver that clearly handles the transition of state. Special care must be taken to assure that the temporal indices of variables within the control system program are properly matched.

The engine system has become increasingly important to real time simulations. The engine and engine control system dynamics can interact with the rotor dynamics via the RPM degree of freedom. Traditionally, engine models were very simple, usually just representing the torque dynamics. Contemporary simulations are utilizing increasingly sophisticated engine models incorporating internal engine states to more accurately simulate interactions with the rotor system (Ref. 3).

Aerodynamic Model. To simulate the aircraft in all flight regimes, a fairly comprehensive, total force aerodynamic model is usually required. The model should separate the various airframe components, such as fuselage, wings, empennage, and any other attachments. This separation allows the inclusion of local effects of the airstream such as rotor wake, wing downwash, air turbulence, and other disturbances. The most common approach to modeling the aerodynamic forces and moments is with the use of tables of aerodynamic coefficients.

The data is normally produced from wind tunnel tests where lift, drag, and pitch moments are measured over a range of angles of attack, sideslip, and if appropriate, Mach number. This basic data, in a variety of combinations and for various airframe components makes up the "aero data" for the simulation model.

The aero data is usually in dimensionless coefficient form in the wind axis system. These coefficients are computed by taking the force (lift or drag) and dividing by the term $\frac{1}{2}\rho v^2 A$, where ρ is the air density, v is the velocity relative to the air mass, and A is a reference area (such as wing area). The moments are divided by $\rho v^2 A l$, where l is a reference length (such as wing cord). The wind axis differs from the body axis by rotations through the angles of attack and sideslip. It must be noted that many variations exist for the definitions of the coefficients and the of wind axis. A specific model must define the conventions being used.

The typical scheme for modeling the "aero" is to define the local velocities, Mach number, and angles of attack and sideslip. If the horizontal stabilizer is taken as an example; the local velocity would be the velocity at the aircraft CG, modified by body rotational rates and wake effects from the wing, fuselage, and/or rotor system. The local angles of attack and sideslip are defined with respect to the local velocity components. Aerodynamic coefficients are found (interpolated) in the database using, for instance, the angle of attack and elevator deflection. The forces are found by multiplying the coefficients by the non-dimensionalizing factors. The forces and moments are transformed through the angles of attack and sideslip to the body axis. In some cases, the equations for wind axis forces and moments can become quite complicated as various influences are modeled. Sideslip, rate of change of downwash, and airstream blockages are examples. By modeling each airframe component separately, these special considerations can easily be included. Once all the aerodynamic forces and moments for each subsystem are modeled, they can be summed at the aircraft CG.

Helicopter Main Rotor Model. The rotor system model is usually the most complicated module of the entire simulation mathematical model. Historically, the rotor model was severely limited in complexity due to computational difficulties of running the model in real time. With today's low cost, high speed computing systems, modelers are continually adding detail and new complexities to the model. The rotor modeling methods are too vast to examine in any detail here, but general aspects will be discussed.

There are two modeling approaches used extensively by simulations; the blade element method which is dominant in contemporary simulations, and the tip-path-plane method which has a considerably reduced computational requirement (Refs. 4, 5, 6). For the blade element approach, the rotor blade dynamics are represented by degrees of freedom in a rotating coordinate system that spins with the rotor itself. The forces and moments are defined in this rotating frame and summed at the hub. The tip-path-plane approach makes use of a Fourier transformation to represent the rotor dynamics in a non-rotating coordinate system. Essentially, the rotor is treated as a tilting disk and forces and moment are described in this non-rotating frame. The discussion that follows will describe the general approach for the blade element modeling method.

The rotor model can be divided into several submodules; the rotor induced velocity or inflow, blade dynamics, rotor forces and moments, and various coordinate transformations. The inflow model is the least exact submodule of the rotor model. The simple models in use today typically have a uniformly distributed component that comes from momentum theory plus harmonically distributed components based upon the periodic moments acting on the rotor disk. It is also common to impose a "dynamic" behavior to the inflow by the use of empirically derived time lags.

The blade motions are represented by complex dynamic equations that are non-linear and have periodic coefficients. The blades move as rigid bodies about a specific hinge or bearing arrangement. Various elastic modes of blade motion can also be represented. From these equations the blade velocities and displacements are known. Combining the blade state with the local velocities (airframe, rotor, and inflow), the blade loads can be described. In the blade element model, the blade aerodynamic forces and moments are defined in a manner similar to other areas of the helicopter aerodynamic model (described above). The blade is divided into a number of radially distributed segments (elements). For each segment, the local velocity and angle of attack are found. Tabulated values of section lift, drag, and pitch moment are interpolated to find the aero forces and moments at each segment. The forces and moments are summed along each blade and for all blades of the rotor.

There are a number of additional coordinate systems that can come into play with the rotor model. The axis system conventions differ depending on the physical arrangements of a particular rotor design. Typically there is an axis system that is a body axis aligned with the rotor shaft. There are various rotating axis systems that progress from the hub, across the blade hinge points, and out to the blade segments. Finally, there is a wind axis system at each blade segment where the aerodynamic forces and moments are defined. As one would imagine, it is absolutely necessary to carefully define each axis system and the transformations from one to another in the model.

With the availability of high speed, parallel computer systems, new areas of rotor system modeling for simulation purposes have become possible. Modelers are adding elastic modes to the blade dynamics and dynamic wake models to achieve a much higher fidelity representation of rotor inflow (Refs. 7, 8). Combined, these advancements make possible a real time, aeroelastic rotor model.

Tail Rotor Model. The mathematical model of the helicopter tail rotor is usually much simpler than that of the main rotor. Modern high speed computer systems allow tail rotor models of greater complexity, but the benefits are not significant for most purposes. For most cases it is sufficient to model the thrust and required torque of the tail rotor.

A common method for representing the tail rotor in a simple fashion is with the use of the Bailey theory (Ref. 9). This is a closed form approach that uses parameters that only depend on blade pitch and the inflow ratio. A set of "t" coefficients are computed that are in turn used to calculate thrust and torque required. The thrust is resolved into body axis forces and moments. Torque can then be made available to a drive train model.

Environmental Model. The simulation requires certain characteristics about the airmass. The aero and engine models require air density, pressure, and temperature. The aero model requires airmass velocities including steady or variable winds and gusts, wind shears, and air turbulence. The first items are fairly standard and can be referred to as the atmospheric model. Simulations can use the standard atmospheric databases, "hot day" databases, or any collection of special ones (Ref. 10, 11). The databases are usually interpolated with aircraft altitude to yield density, pressure, and temperature.

The velocity variations of the airmass are best modeled in two parts; the lower frequency, deterministic gusts, shears, and steady winds, and the higher frequency, random turbulence. The deterministic velocities are defined in the fixed (inertial) frame of reference and can be modeled as functions of time and spatial coordinates. A reasonable method of implementing these winds is to add them to the inertial velocities and then rotate them to the body axis. These new definitions for the body axis translational velocities, u, v, and w, make them "relative velocities", i.e., relative to the wind. They are the velocities upon which aero forces and moments are based.

Air turbulence is usually represented by a statistical model (Ref. 2). These models typically assume the velocity variation to be random, isotropic, and with negligible cross-correlation between components. These models also assume Taylor's hypothesis, or the frozen field concept, which states that a patch of turbulence is constant or "frozen in time". Grossly stated, each component of turbulence is modeled by running a normally distributed noise signal through a shaping filter or correlation function. The Dryden and von Karman models are examples. These models use a characteristic scale length which can be taken as a wing span or a rotor radius. The turbulence models have been commonly used for fixed wing aircraft and have been "forced" into rotorcraft models. Because of the rotating blades, however, these models were not really appropriate to rotorcraft. Recent work (Ref. 12) has attempted to improve the modeling for the case of rotational sampling of the turbulent field.

SUMMARY REMARKS

In this chapter a general discussion of the type of mathematical model used in a real-time, flight simulation was presented. The mathematical model was defined as a description and specification of the aircraft's dynamical behavior. This behavior comprises the various motions of the airframe and the states and performance of the various subsystems, such as the engine, landing gear, and avionics. The model is best presented by mathematical equations, schematic diagrams, logic diagrams, tables and graphs of data including geometry, aerodynamic coefficients, gain schedules, etc. These engineering equations, diagrams, and data comprise the mathematical model.

It is recommended that the approach to math model development include modularity and standardization. Standard conventions should be defined and adhered to by all modules. The information interfaces among modules must be defined as well as any interfaces to the "outside world". Modification and maintenance of the model will be much more efficient with this approach.

The discussion of simulation mathematical modeling included the general equations of motion and the specific aircraft subsystems. The general equations of motion that form the core of the simulation were developed along with all the equations necessary to predict the aircraft's state when the external forces and moments are known. Finally, some of the typical aircraft specific modules were examined to give the reader some idea of what might be included in contemporary flight simulation models.

SYMBOLS

D_N, D_E, D_D	aircraft position in the local axis system, ft
$\vec{F}$	total external force acting on the aircraft, lb
F_x, F_y, F_z	components of the total external force acting on the aircraft in the x, y, and z directions, respectively, lb
F_N, F_E, F_D	components of the total external force acting on the aircraft in the local axis system , lb
g	acceleration due to gravity, ft/sec/sec
$\vec{H}$	the angular momentum, ft-lb-sec
I_{xx}, I_{yy}, I_{zz} I_{xy}, I_{xz}, I_{yz}	moments and products of inertia, lb-sec2-ft
$\hat{i}, \hat{j}, \hat{k}$	orthogonal triad of unit vectors aligned with the x, y, and z body axes, respectively
L, M, N	components of total moment about the x, y, z directions, ft-lb
m	the aircraft mass, slugs (lb-sec^2/ft)
$\vec{M}$	total external moment acting at the center of mass, ft-lbs
p, q, r	the body axis rotational velocities about the x, y, and z axes, respectively, r/sec
u,v,w	the body axis translational velocities in the x, y, and z directions, respectively, ft/sec
$\vec{v}$	the absolute velocity of the center of mass, ft/sec
V_N, V_E, V_D	aircraft velocities in the local axis system, ft/sec
x, y, z	the body axis; origin at the aircraft CG, the x-axis is pointed forward out the nose; the y-axis is pointed out the right side of the aircraft, and the z-axis is directed down through the underside of the aircraft
X_L, Y_L, Z_L	the local axis; origin at the aircraft's CG with a fixed orientation, the X_L-axis always points north, the Y_L-axis points east, and the Z_L-axis points down
X_E, Y_E, Z_E	the inertial axis; origin fixed at some point on the earth's surface, X_E-axis always points north, the Y_E-axis points east, and the Z_E-axis points down

α	aircraft angle of attack, rad
β	aircraft sideslip angle, rad
ϕ, θ, ψ	Euler angles describing the aircraft's orientation; ϕ is the roll angle, θ is the pitch angle, and ψ is the yaw angle, radians
$\vec{\omega}$	the aircraft's total angular velocity , r/sec

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