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MINIMUM-COMPLEXITY HELICOPTER SIMULATION MATH MODEL PROGRAM

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ABSTRACT

An example of a minimal complexity simulation helicopter math model is presented. Motivating factors are the computational delays, cost, and inflexibility of the very sophisticated math models now in common use. A helicopter model form is given which addresses each of these factors and provides better engineering understanding of the specific handling qualities features which are apparent to the simulator pilot. The technical approach begins with specification of features which are to be modeled followed by a build-up of individual vehicle components and definition of equations. Model matching and estimation procedures are given which enable the modeling of specific helicopters from basic data sources such as flight manuals. Checkout procedures are given which provide for total model validation. A number of possible model extensions and refinements are discussed. Math model computer programs are defined and listed.
FOREWORD

This report was prepared by Manudyne Systems, Inc., for the U. S. Army Aeroflightdynamics Laboratory located at Ames Research Center. The Contract Technical Monitors were Ms. Michelle M. Eshoew and Mr. Christopher L. Blanken.

The Manudyne project engineer was Mr. Robert K. Heffley and the math model development effort was conducted by Mr. Marc A. Mnich.
MINIMUM-COMPLEXITY HELICOPTER SIMULATION MATH MODEL PROGRAM

I. Introduction

A. Background

Over the past decade there has been a trend toward increasingly complex simulator math models. Part of this has been a result of flight control system sophistication and attention toward a number of aerodynamic factors, including aeroelastic effects. Another reason is the availability of large, high speed mainframe and mini-computers. Perhaps the most distressing reason for increasing complexity is the general hesitance to determine precisely the degree of complexity really needed for a given application. Unfortunately this trend has some serious implications for and effects on the management and operation of flight simulation.

The question being considered is really one of value versus cost. The value must ultimately be expressed as the utility of a math model to provide necessary features which can be appreciated by the simulator pilot. It is expected that as a function of complexity, this quality approaches a fairly flat asymptote with some reasonable level of complexity. The other side of the coin is the cost of math model development and checkout, also as a function of complexity. Unfortunately this function can be expected to increase exponentially. Both these relationships are sketched in Figure 1. The obvious question for the simulator user is at what level of model complexity do these two curves cross.

![Diagram of Cost vs. Model Complexity]

Figure 1. Tradeoff of Math Model Cost with User Utility.
Experience typical of those described in References 1 and 2 has shown that math model complexity alone does not automatically provide effectiveness in handling qualities simulations. Rather, there can be distracting factors which work counter to simulation objectives. Ultimately, limited resources prevent one from realizing the full potential of an overly complex simulator math model. Other limitations can be a lack of flexibility in modeling and restricted clarity in the cause and effect relationships between model parameters and features. These shortcomings raise questions about the value of complexity in helicopter math models and are a motivation to consider simpler models.

1. Computational Delays

Computational lag and delay is a particularly important problem resulting from model complexity. As complexity grows, computational delay associated with the math model code increases and, in turn, compounds overall visual system delay. Computer speed is limited by the hardware and software system being used and cannot be easily changed.

The result of the delays imposed is reduced fidelity. NASA Ames, for example, employs both a Xerox Sigma 8 and CDC 7600 for their Vertical Motion Simulator (VMS). Using the currently implemented ARMCOP helicopter math model (Reference 3) and the faster of the two computers (CDC 7600), the computational delay is about 25 milliseconds. The Sigma may require 60 to 75 milliseconds to cycle. The former speed is acceptable but CGI delays of about 100 milliseconds can still remain. The problem of delay has been addressed and software added to alleviate it. Thus more complexity has been added to correct a problem originally caused by complexity. This is not as effective as preventing the problem by simplifying code or using a faster computer.

2. Cost of Resources

As model complexity and the amount of computer code grows, so does the time and effort required to implement, check, and debug the code. The time available to do these is often limited and can affect overall math model fidelity if neglected. ARMCOP, for example, has several thousand lines of code. In checking and debugging code in large programs, a certain number of errors will go undetected, and the more code there is, the more likely errors will persist.

In addition to checking and debugging code there is the task of determining model parameters needed to represent a specific aircraft. The time and effort required to thoroughly check the model against the real aircraft can be an expensive part of the simulation being run. Math models employing look-up tables can have hundreds of parameters which need to be set and
confirmed. Since some of these values are estimations, an iterative process may be required. Limits on time and manpower may restrict this process and the fidelity of the model.

Validation of the math model equations (as opposed to math model code) is also a process which may require iteration as the model is changed. It is possible that errors in the math model will exist even as the model is being used in simulation. Again, the number of errors which exist and the time required to fix them is a function of the complexity of the model. Time and manpower restrictions will limit the ability of the users to find and correct these errors and thus degrade the fidelity of the model. In order to guarantee that a model is completely correct, all parts of the model must be exercised. Lookup tables, for example, require that all numbers in the table be verified as well as checked for discontinuities. All equations in the model need to be checked to ensure they are theoretically sound. With complex code, it is unlikely that all of the model will be checked as thoroughly as necessary and errors can persist in actively used models for long periods of time before they are ever noticed or corrected.

ARMCOP, for example, still contains at least a serious error affecting maneuvering flight even though the model has seen wide use. This error involves a large speed loss in sustained turns. Although detected, this problem has not been corrected because of time constraints. Rather it has been "patched up" with flight control system modifications. Again, complexity is added to fix a problem itself arising from model complexity.

3. Inflexibility

There is an inherent tradeoff between complexity and flexibility in models of dynamic systems. As more components or features are added to a model, it becomes increasingly difficult and expensive to perform other modifications. One measure of the flexibility of a model is its adaptability to new computer systems and languages or to changes in the code. Large sets of code are limited to large computer systems. ARMCOP, for example, requires the use of a mainframe system. In order to work with the model, one must have access to such facilities.

Once code has been implemented on a machine, it must be checked and debugged. Modifications for debugging may require recompilation and require significant amounts of time for large code. Most of these changes are made before the code is used for actual simulation, but it is possible that changes are needed during simulation. Even simple changes can consume enough time to hamper productivity. Changing a single parameter in ARMCOP, for example, requires a minimum down time of 20 minutes.

The ability to add, remove, or modify efficiently the dynamic characteristics of a model is another measure of its flexibility. It may be desirable, for example, to have a
helicopter simulation without rotor cross coupling. A model such as ARMCOF, in which cross coupling is inherent, does not allow easy removal of this feature. In fact, it would probably be "removed" by adding control system features to null the coupling thus further increasing the complexity of the model. The emphasis in modeling should be with the efficiency of the model while maintaining adequate fidelity.

4. Indirectness of Cause and Effect Relationships

The ability to see the relationship between model parameters and model response features is decreased with complexity. This relationship is important to handling qualities simulation work for two reasons. First, is the need to easily make changes in model features. Second is the need to trace errors which appear in the response modes of the model. These are fundamental to working effectively with the model. In order to modify response features, one must know what parameters are responsible for those features and how to change them. In complex models, individual parameters tend to become coupled to many features at once making it difficult to change features independently.

B. Merits of Considering a Simple Math Model Form

It would appear that there are compelling benefits for general reductions in the levels of complexity exemplified by math models such as ARMCOF and GENHEL. This leads us to consider ways to find a compromise between math model complexity and simulator utility. At one extreme are the highly complex models which attempt to achieve effectiveness through high computational fidelity. As mentioned, these models encounter practical limits which not only hamper fidelity but also reduce their flexibility and clarity between parameters and features. At the other extreme are models such as the linearized stability derivative form which are easier to manage but which may lack fidelity or be restricted to a small operating envelope.

The merits of a "compromise" models form will thus be cost and quality benefits derived from the achievement of specific fidelity features through minimal software, i.e., program instructions.

1. Cost

The cost benefits will accrue through minimizing labor required to quantity and checkout the math model implementation. Development of even modest math models typically involve more than one man year of labor. If this process can be shortened to less than one man-month, the period envisioned for the proposed form, then great savings clearly can be realized.

Simulator math model software checkout can also require substantial effort. However, this is often simply limited by
time available and the job might not actually be completed prior to simulator use. Again the aim is to realize greatly reduced checkout time through software reduction and to make a comprehensive checkout feasible within a short period of time.

2. Quality

The quality benefits come from confidence that specific features needed for effective simulation are represented and that they are correct. Here quality arises from the fact that implementation and checkout tasks which should be done are, in fact, done. In a real sense, quality follows the degree of manageability afforded by the simulator software.

3. Engineering Understanding

One of the most important benefits to be derived from a minimum-complexity math model is in the potential for more clearly understanding cause and effect relationships. For example, if a particular kind and amount of cross-coupling is desired, then how does one achieve it through adjustment of math model parameters? It is possible by having a close, easy-to-follow connection between the physical component representation and the resulting physical response features.

An important value of engineering understanding is the ability to make model adjustments or refinements in a direct, efficient manner.

C. Model Attributes to be Considered

1. Simulator Application

It should be stressed, here, that in this case the goal of the math model is to be an effective tool for simulation. Model fidelity alone is not the solution to simulator effectiveness. Rather, it is the ability of the model to produce the desired results for the given application. Besides having adequate fidelity, the model must also be affordable, manageable, easily modified and checked, and have a clear cause and effect relationship between parameters and features.

2. Handling Qualities Application

Thus we are motivated to turn to a simple model with these qualities for helicopter handling qualities simulation which can be a more effective tool than existing models. Specifically, the purpose here is to propose a minimum-complexity model format suitable for helicopter handling qualities simulation.

It should be remembered that most handling qualities investigations involve examination of fairly crude and simple parameters such as time constants, damping ratios, or static gains. Furthermore the precision with which evaluation pilots
can see such changes is often disappointingly low. Thus it is not reasonable to expect that high math model resolution is really crucial. If a pilot cannot actually observe or be influenced by certain math model effects then those effects should probably be considered as excessive complication.

3. Full Flight Envelope Operation

The model will be nonlinear and will apply to the full operating range of a real helicopter including rearward as well as forward flight, sideward flight, hover, and transition from hover to forward flight. The model will include first order flapping degrees of freedom and all rigid body degrees of freedom. Not included will be the higher order flapping modes and any structural modes as they are well beyond the frequency range of interest for handling qualities.

4. Modularity

The form of the model will be modular. This will allow the flexibility of adding additional rotors if desired as well as any other lifting surfaces. Any combination of components can be combined including models of pilots and control systems making the model adaptable to a variety of helicopters and subsystems. The full utility of the model format will become apparent as the structure of the model is described in more detail.

5. Microcomputer Adaptability

The math model form will be compatible with microcomputer use, at least on a non-real-time basis. It has been found that math model development and checkout can be done to a large extent on small, inexpensive desktop microcomputers. This of course demands that the software be reasonable compact.

D. Report Organization

The presentation to follow will consist of four parts: (i) approach to modeling, (ii) matching and estimation procedures, (iii) checkout procedures, and (iv) extensions and modifications of the model. In addition various detailed information will be contained in appendices.

1. Modeling Approach

In the first section, the modeling approach will be described in order to establish the theoretical foundation for the model. This will also be useful for understanding, modifying or extending the model and for its effective use as a simulator tool. In addition, a description of the features and components of this specific model is given. The model is used to represent a Bell AH-1S Cobra. All parameters and variables from this aircraft are provided here along with the actual code. The
sample version will show the extent of the code in terms of number of parameters, number of lines of code, number of computations, etc. and will be compared to an ARMCO-P version of the same aircraft.

2. Matching and Estimating Procedures

In the next section, the matching and estimating procedures used to obtain model parameters are described. The sample version of the AH-1S is used as a specific example. The model is then exercised and the estimated parameters varied in order to tune the model to fit performance data.

3. Checkout Procedures

The third section describes several methods of checking the math model code. The size of the model and the modular format are conducive to efficient checking. Methods are then presented for verifying the math model equations and are illustrated using the sample version.

4. Model Extensions and Refinements

Finally, in the last section, possibilities for extending or modifying the model are introduced to demonstrate the flexibility of the model format. The potential for a much improved level of simulation effectiveness using these extensions and modifications is revealed and explained in terms of the approach taken to the modeling process.
11. Technical Approach

A. Specification of Desired Math Model Features

The approach to modeling will begin with a list of desired features. This list will serve as a specification upon which to formulate a minimum-complexity model containing only those components and equations directly responsible for the desired features. The model will be customized to the problem being studied.

We shall assume that the model is intended for handling qualities simulation and that the features to be included in the model should be features visible to a pilot. The response features to be contained are listed in Table 1.

Table 1. Desired Response Features

1. First-order flapping dynamics for main rotor.
2. Main rotor induced velocity computation.
3. All rigid-body degrees of freedom.
4. Realistic power req’ts over the desired flight envelope.
5. Rearward and sideward flight without computational singularities.
6. Hover dynamic modes:
   Longitudinal and lateral hover cubics
   Rotor-body coupling with flapping
7. Forward flight dynamic modes:
   Short period
   Phugoid
   Roll mode
   Dutch roll
   Rotor-body coupling with flapping
8. Dihedral effect.
9. Correct transition from hover to forward flight.
11. Correct power-off glide for min rate of descent and max glide.
1. **First-Order Flapping**

   It has been shown in Reference 4 that rotor flapping can couple with rigid-body modes in regions which affect handling qualities. This occurs in the lower frequency or "regressing flapping" modes. However, this effect can be modeled with a first-order flapping equation in each of the pitch and roll axes.

   The time constant involved in the regressing flapping mode is directly proportional to the product of rotor angular velocity and Lock number. Thus only the commonly available rotor mass and geometric parameters are needed.

   The actual flapping response is modified by coupling with the fuselage at the hub restraint. Since this involves the classical rigid body modal response, it will be further discussed under items 6 and 7 below.

   The feature of flapping which is most important to a pilot-in-the-loop simulation is the apparent control lag following cyclic input. This lag is in effect the time required to precess the tip path plane to a new orientation. A typical value for the effective lag is about 0.1 sec—significant because it is comparable to the pilot's own neuromuscular lag.

2. **Main Rotor Induced-Velocity Computation**

   A particularly important feature of a helicopter is the relationship among thrust, power, and airspeed. This relationship arises from the induced-velocity of air passing through the rotor disc.

   There are a number of complicating factors, but to a reasonable first-order approximation induced-velocity effects can be modeled with a classical momentum theory model wherein thrust and induced-velocity interact in an aerodynamic feedback loop. Computation is complicated, however, because this feedback is highly nonlinear.

   Another aspect of the induced-velocity is its effect on adjacent surfaces. The rotor induced-velocity field impinges on the wing, horizontal tail, and fuselage and varies with airspeed and flight path direction.

3. **Rigid-Body Degrees of Freedom**

   Normally, six rigid-body degrees of freedom are needed for useful manned simulation. Pilot workload arises from constant attention to roll, pitch, and yaw as well as translation fore-and-aft, to the side, and vertically. Only under special conditions might one desire to eliminate one of these via, for example, the assumption of perfectly coordinated forward flight.

4. **Power Requirements Over Flight Envelope**
A common source of real aircraft data appropriate for verifying a math model is performance data in terms of power required for various trim conditions. The power or torque required is immediately obvious and important to a pilot and varies substantially from hover through transition and finally in forward flight.

Power requirements can be easily computed once main and tail rotor induced velocities are established.

5. Rearward and Sideward Flight

In a full-flight-envelope model involving circulation lifting surfaces, computational singularities can exist, depending upon the model form used. These singularities come from trigonometric functions for angle of attack, sideslip, etc., but are avoided in this model by using a quadratic lift coefficient method. For this technique, forces for lifting surfaces are computed using quadratic coefficients multiplied by the squares of velocity components so that negative velocities cannot cause singularities. No explicit computation of angle of attack or sideslip is needed and, indeed, should be completely avoided.

6. Hover Dynamic Modes

Hovering flight is characterized by similar dynamics in each the pitch and roll axes, including sets of high and low frequency response modes. In addition, the yaw axis contains a predominant yaw damping mode. These dynamics can couple with regressing flapping dynamics. All are apparent to the pilot in operating the aircraft whether trimming, maneuvering, or flying unattended.

Pitch and roll is classically described by the "hover cubic," but this is generally neglects coupling with the rotor which can be important. This is easily computed, however, through inclusion of the flapping dynamics as described earlier.

The phugoid mode for hover results from the combination of dihedral and gravity force. Effective dihedral is particularly apparent in unaggressive sideward flight because the pilot must continually add lateral control as sideward velocity increases.

7. Forward Flight Dynamic Modes

In forward flight the dominant rigid body dynamics of a helicopter resemble those of a conventional fixed-wing airplane and include short-period, phugoid, dutch roll, and spiral modes. There is also likely to be significant coupling with flapping dynamics.

8. Correct Transition from Hover to Forward Flight
Transition effects are an important part of the piloting task when accelerating from hover into the forward flight region.

These effects are a combined result of a "dihedral effect" in the x-axis and the varying rotor downwash effect on the horizontal tail.

9. Effects of Rotor RPM Variation

Rotor RPM can affect helicopter dynamics in a number of ways, including thrust, flapping response, and heave damping.

The effects of rotor speed variation are tied, however, to the rotor-engine-governor combination. For a number of applications it may be sufficient to assume a constant rotor RPM. This will be done here.

10. Cross Coupling

A variety of cross coupling effects can be present in helicopters. Some of these such as collective-to-yaw coupling are easy-to-see first order phenomena. These are generally inherent in the basic dynamics if reasonable first-principles thrust and rotor models are used.

Other coupling effects are more subtle and should be added only where desired by the simulator user. These can be inserted directly in the equations of motion as coupling terms arising from both angular and translational velocity components or controls.

11. Correct Power-Off Glide

Helicopters, like fixed-wing aircraft, need to exhibit reasonable performance when power is reduced. This can be a highly complex issue if ring vortex rotor states are included. Generally, however, handling qualities investigations can be conducted using only the normal thrust model described above but tailoring the full-down collective pitch and aerodynamic drag to yield realistic forward-velocity glide characteristics.

B. Component Build-Up

With a specification of desired features, essential model components can then be chosen. These components contain the mechanisms which provide forces and moments, power dissipation, stability and control, and rotor dynamics.

The six components are considered necessary to provide all of the above response features are shown in Figure 2. Table 2 lists these components along with the physical elements of each component and the response features which result from them. The components and their physical elements are described and discussed individually below.
Figure 2. Basic Helicopter Math Model Components.
Table 2. Details of Component Build-Up.

<table>
<thead>
<tr>
<th>Components</th>
<th>Physical features</th>
<th>Response features</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.) Main rotor:</td>
<td>Thrust</td>
<td>1st order flap-ping</td>
</tr>
<tr>
<td></td>
<td>Torque</td>
<td>Power required</td>
</tr>
<tr>
<td></td>
<td>Induced velocity</td>
<td>Trim</td>
</tr>
<tr>
<td></td>
<td>Tip path plane lag</td>
<td>Phugoid</td>
</tr>
<tr>
<td></td>
<td>Induced power</td>
<td>Short period</td>
</tr>
<tr>
<td></td>
<td>Profile power</td>
<td>Dihedral</td>
</tr>
<tr>
<td></td>
<td>L1 - M1 = 0</td>
<td>Pitch mode</td>
</tr>
<tr>
<td></td>
<td>Lp - Ma = 0</td>
<td>Roll mode</td>
</tr>
<tr>
<td></td>
<td>Constant RPM</td>
<td>Min x-coupling</td>
</tr>
<tr>
<td></td>
<td>Incremental Lb1 and Ma1</td>
<td>Power off glide</td>
</tr>
<tr>
<td>2.) Fuselage:</td>
<td>Mass at C.G.</td>
<td>Trim</td>
</tr>
<tr>
<td></td>
<td>Moments of inertia</td>
<td>Power required</td>
</tr>
<tr>
<td></td>
<td>Parasite power</td>
<td>Min x-coupling</td>
</tr>
<tr>
<td></td>
<td>Cross products of inertia = 0</td>
<td>Power off glide</td>
</tr>
<tr>
<td>3.) Tail rotor:</td>
<td>Thrust</td>
<td>Trim</td>
</tr>
<tr>
<td></td>
<td>Torque</td>
<td>Power required</td>
</tr>
<tr>
<td></td>
<td>Induced velocity</td>
<td>Roll mode</td>
</tr>
<tr>
<td></td>
<td>Induced power</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Profile power</td>
<td></td>
</tr>
<tr>
<td>4.) Horizontal tail:</td>
<td>Lift / Stall</td>
<td>Short period</td>
</tr>
<tr>
<td></td>
<td>Exposure to main rotor induced vel.</td>
<td>Trim</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pitch mode</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Power required</td>
</tr>
<tr>
<td>5.) Wing:</td>
<td>Lift / Stall</td>
<td>Trim</td>
</tr>
<tr>
<td></td>
<td>Induced drag</td>
<td>Power required</td>
</tr>
<tr>
<td></td>
<td>Induced power</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exposure to main rotor induced vel.</td>
<td></td>
</tr>
<tr>
<td>6.) Vertical tail:</td>
<td>Lift / Stall</td>
<td>Dutch roll</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roll mode</td>
</tr>
</tbody>
</table>
1. Main rotor

The primary component of this model is the main rotor. It is the main feature responsible for producing characteristics unique to a helicopter, in particular, a vertical thrust vector and an induced-velocity field. Other key ingredients include rotor torque, dihedral effect, and flapping dynamics.

The basis for the model used here is primarily the autogyro theory presented by Glauert in Reference 5 and extended by Lock in Reference 6. The higher-order flapping dynamics as defined by Chen in Reference 7 are simplified according to the model developed by Curtiss and presented in Reference 1.

Thrust and induced velocity are computed assuming a uniform flow distribution. As described earlier, the tip-path-plane orientation (flapping angles) are modeled as simple first order lags giving the main rotor the qualities of a force actuator with a lag.

The main rotor model contributes largely to the power requirement feature of the model. In hover, nearly 80% of total power is required by the main rotor. In forward flight, as much as 60% of total power is absorbed by the main rotor. Induced velocity also accounts for power losses by the fuselage in hover.

Cross coupling in the main rotor can be minimized if desired. Here control cross coupling ($L_{a1}$ and $M_{a1}$) and gyroscopic cross coupling ($L_{q}$ and $M_{p}$) are not included as an essential part of the main rotor model. Some of these effects would be inherent in using a more complete flapping model. But a more useful fact is that such features can be modeled directly in order to achieve the precise effect desired.

The dihedral effect is included through the variables $db_1/dv$ and $da_1/du$ which appear in the first order flapping equations. Values can be computed using first-principles factors consisting of thrust coefficient and tip velocity. The dihedral feature is responsible for the phugoid-like modes in hover and forward flight.

The portion of $L_{b_1}$ and $M_{a_1}$ due to both hinge offset and rotor spring stiffness are included in a separate parameter, $dM/da_1$. Thus, the total flapping stiffness can be directly varied through this one parameter.

Pitch and roll mode time constants are a function of both body pitch and roll damping and rotor tip path plane lag. Control over these time constants can thus be exercised through the flapping lag as well as body aerodynamic damping.

2. Fuselage
The fuselage is represented as a virtual flat plate drag source having three dimensions. The effective aerodynamic center can be located at any position in the body reference frame. It would normally be expected to be near the geometric center.

The fuselage drag model is based on a quadratic aerodynamic form originally found in the hydrodynamics text by Lamb (Reference 7) and used extensively for airship applications by Monk (Reference 8). This form can be easily extended to account for fuselage asymmetries, lifting effects, and lift gradients.

The simple fuselage aerodynamic form presented here provides for drag in forward flight which limits maximum airspeed, drag in sideward flight, and rotor downwash impinging on the fuselage. All three of these effects are related to power losses.

3. Tail Rotor

The tail rotor component is modeled in the same manner as the main rotor except that no flapping degree of freedom is included. In effect, only Glauert's equations apply. However, thrust, induced-velocity, and power effects are correctly modeled. Normal directional control is provided through the tail rotor collective pitch variation.

4. Horizontal Tail

The horizontal tail is assumed to be primarily a lift producer, thus only the normal force component is modeled. This still provides for computation of drag resulting from induced-lift if that is desired. Finally, the effects of aerodynamic stall are included. The geometric location of the horizontal tail in the rotor flow field is used to obtain the local apparent wind component. The location of the horizontal tail provides effective static stability and elevator control.

As with the fuselage aerodynamics, a basic quadratic form is used. Two terms model the effects of camber and circulation lift. One additional term and conditional test is included to model the effect of stall.

5. Wing

The wing component follows the same form as the horizontal tail. In addition, the induced drag is computed in order to obtain the related power-required component which can be significant during sustained-g maneuvering.

6. Vertical Tail
The vertical tail is also similar to the horizontal tail except that it experiences the flow field produced by the tail rotor.
C. Definition of Model Equations

Once the various components of the model are defined, the equations for all the components must be expressed in a way which minimizes code and the number of parameters. The following does so according to the order of the computer program.

1. Main Rotor Thrust and Induced Velocity

The computation of thrust and induced velocity is based on a classical momentum theory equation, but with a special recursion scheme which yields a very quick convergence. The block diagram showing the thrust and induced velocity equations is given in Figure 3.

Figure 3. Main Rotor Thrust and Induced-Velocity Block Diagram.

The recursion relationship is based on breaking the thrust-induced velocity loop at the induced-velocity node and iterating on a solution for thrust followed by induced-velocity. This yields a fast convergence with a fixed number of iterations—about 5 is sufficient.
\[ T = (W_b - v_1) \frac{R \text{abcR}}{4} \]

\[ v_1^2 = \frac{\dot{v}^2}{2} + \frac{T^2}{2A} - \frac{\dot{v}^2}{2} \]

where

\[ W_r = W_a + (a_1 + i_5) U_a - b_1 V_a \]

\[ W_b = W_r + \frac{2}{3} R [ \text{col} + \frac{3}{4} \text{twist} ] \]

\[ \dot{v}^2 = \dot{u}_a^2 + \dot{v}_a^2 + W_r(W_r - 2v_1) \]

\[ A = \pi R^2 \]

Once induced velocity for the main rotor has been computed, one can compute the longitudinal and lateral dihedral effects of the main rotor which are, in turn, dependent on induced velocity:

\[ \frac{db_1}{dv} = \frac{da_1}{du} = \]

The main rotor parameters needed for these equations are:

\[ d^m_r, \text{ horizontal distance of hub from c. g.} \]

\[ h^m_r, \text{ hub height above the c. g.} \]

\[ R, \text{ rotor radius.} \]

\[ \text{abcR, product of lift slope, number of blades, chord, and radius.} \]

\[ , \text{ effective blade twist.} \]

\[ , \text{ main rotor angular rate.} \]

2. Tail Rotor Thrust and Induced-Velocity
Thrust and induced velocity for the tail rotor is computed in the same manner as for the main rotor except that no flapping effects are included.

The parameters which define the tail rotor effects are:

\[ d_{tr}, \text{distance of tail rotor from c.g.} \]

\[ h_{tr}, \text{height of tail rotor above c.g.} \]

\[ R_{tr} \]

\[ (abcR)_{tr}, \text{product of lift slope, number of blades, chord, and radius.} \]

\[ \omega_{tr}, \text{tail rotor angular rate.} \]

3. Fuselage Geometry and Drag

Profile drag forces are computed for the fuselage in the x-, y-, and z-axes. These drag forces can constitute a significant portion of the overall power required and thus must be computed prior to main rotor torque. The forces are computed at the center of pressure located at the point (X.FUS, Y.FUS, Z.FUS) relative to the center of gravity.

Fuselage drag forces are computed using a "quadratic aerodynamic form." In this case forces are expressed as a summation of terms formed by the product of translational velocity components in each axis. The constants in each term are the effective flat plate drag.

\[
\begin{align*}
W_{a}^{fus} & = W_{a} + \nu_{l} & \text{local } w\text{-velocity} \\
X_{aero}^{fus} & = \frac{\rho}{2} X_{uu}^{fus} U_{a} \cdot U_{a} & \text{drag component} \\
Y_{aero}^{fus} & = \frac{\rho}{2} Y_{ww}^{fus} V_{a} \cdot V_{a} & \text{side-force component} \\
Z_{aero}^{fus} & = \frac{\rho}{2} Z_{ww}^{fus} W_{a}^{fus} \cdot W_{a}^{fus} & \text{downwash component}
\end{align*}
\]
Moments due to the drag forces relative to the center of gravity are computed.

The parameters required for the fuselage are:

\( d_{\text{fus}} \), distance of fuselage a. c. from c. g.

\( h_{\text{fus}} \), height of fuselage a. c. from c. g.

\( x_{\text{fus}} \), effective flat plate drag in x-axis

\( y_{\text{fus}} \), effective flat plate drag in y-axis

\( z_{\text{fus}} \), effective flat plate drag in z-axis

4. Horizontal Tail Geometry and Lift

The horizontal tail is modeled in terms of a quadratic aerodynamic form for airfoils.

The first step in computing the lift on the horizontal tail is to determine whether the surface is immersed in the rotor downwash field. This will influence the local vertical velocity vector.

The next step is to check for aerodynamic stall by comparing the force computed above with the maximum achievable at the same airspeed.

\[
W_0^M \triangleq W_0 + v_i \quad \text{local w-velocity}
\]

\[
Z_{\text{aero}}^M = \frac{\rho}{2} \left( Z_{uu}^M U_0 U_a + Z_{ww}^M U_0 W_0^M \right) \quad \text{normal force}
\]

\[
> \frac{\rho}{2} Z_{\text{min}}^M U_a U_0 \quad \text{stall condition}
\]
Pitching moment due to the horizontal tail is computed based on the location of the aerodynamic center relative to the center of gravity.

The parameters required for horizontal tail effects are:

- \(d_{ht}\), distance of horizontal tail from c. g.
- \(h_{ht}\), height of horizontal tail from c. g.
- \(Z_{uu}\), aerodynamic camber effect
- \(Z_{uw}\), lift slope effect
- \(Z_{min}\), stall effect

5. Wing Geometry and Lift

The wing is treated in the same manner as the horizontal tail. It is first checked for exposure to main rotor downwash and then for stall. For the wing, induced drag is computed in order to determine the power loss due to this effect. Lift and pitching moment for the wing are also computed.

\[
W_{wa}^{\text{wing}} = W_a + V_l \\
Z_{\text{zero}} = \frac{\rho}{2} (Z_{uu} U_\alpha U_a + Z_{uw} U_\alpha W_a^{\text{wing}}) \\
> \frac{\rho}{2} Z_{\text{min}} U_\alpha U_a
\]

The power due to the induced drag of the wing is computed based on the product of force and velocity in the x-axis.

The parameters required for wing effects are:

- \(d_{\text{wing}}\), distance of wing from c. g.
\( h_{\text{wng}} \), height of wing from c. g.

\( z_{\text{uu}}^{\text{wng}} \), aerodynamic camber effect

\( z_{\text{uw}}^{\text{wng}} \), lift slope effect

\( z_{\text{min}}^{\text{wng}} \), stall effect

6. Vertical Tail Geometry and Lift

The vertical tail is treated the same as the other lifting surfaces except that it is assumed out of main rotor downwash. Sidewash from the tail rotor is neglected.

\[
\begin{align*}
V_{\alpha}^{\text{vt}} & \triangleq V_{\alpha} + V_{\text{tr}}^r \\
\gamma_{\text{aero}}^{\text{vt}} & = \frac{\rho}{2} \left( V_{\alpha}^{\text{vt}} U_{\alpha} U_{\alpha} + \gamma_{\text{uv}}^{\text{vt}} U_{\alpha} V_{\alpha}^{\text{vt}} \right) \\
& > \frac{\rho}{2} \gamma_{\text{min}}^{\text{vt}} U_{\alpha} U_{\alpha}
\end{align*}
\]

The parameters required for vertical tail effects are:

\( d^{\text{vt}} \), distance of vertical tail from c. g.

\( h^{\text{vt}} \), height of vertical tail from c. g.

\( \gamma_{\text{uu}}^{\text{vt}} \), aerodynamic camber effect

\( \gamma_{\text{uv}}^{\text{vt}} \), lift slope effect

\( \gamma_{\text{min}}^{\text{vt}} \), stall effect
7. Total Power Required

Total power due to the main rotor, tail rotor, wing, and miscellaneous effects are summed giving the total power output by the engine.

Total power required = \( p^{\text{mr}} + p^{\text{tr}} + p^{\text{fus}} + p^{\text{wng}} + p^{\text{climb}} \)

\( p^{\text{mr}} = \frac{p^{\text{mr}}}{\text{induced}} + \frac{p^{\text{mr}}}{\text{profile}} + \frac{p^{\text{mr}}}{\text{accessories}} \)

(Note: An estimate of power required for accessories can be found in Reference 9.

\( \frac{p^{\text{mr}}}{\text{induced}} = \)

\( \frac{p^{\text{mr}}}{\text{profile}} = \)

\( p^{\text{tr}} = \frac{p^{\text{tr}}}{\text{induced}} = \)

\( p^{\text{fus}} = \frac{1}{2} X^{\text{fus}} U_a^2 + \frac{1}{2} Y^{\text{fus}} V_a^2 + \frac{1}{2} Z^{\text{fus}} (W_a - V_i)^2 \)

\( p^{\text{wng}} = \)

\( p^{\text{climb}} = m g h \)

8. Summation of Force and Moment Equations

The first order effects of all components are summed in three force equations and three moments equations. The force due to gravity rotated through theta and phi are also included here:

\( X = \)

\( Y = \)

\( Z = \)

\( L = \)
\[ M = \]

\[ N = \]

The equations of motion are expressed in terms of body axis accelerations so that they may be directly integrated.

9. Integration and Axis Transformation

As discussed in Reference 10 the algorithm used for numerical integration of states should be carefully chosen to minimize digital effects.

The body accelerations are integrated using a second order Adams method:

\[ \mathbf{v}_{n+1} = \mathbf{v}_n + DT \left( 1.5 \mathbf{a}_n - 0.5 \mathbf{a}_{n-1} \right) \]

These body velocities are then converted to earth relative velocities using a common Euler angle direction cosine transformation.

Finally, the earth velocities are integrated to obtain earth positions using a trapezoidal integration method:

\[ \mathbf{x}_{n+1} = \mathbf{x}_n + DT \left( 0.5 \mathbf{v}_n + 0.5 \mathbf{v}_{n-1} \right) \]
10. Summary of Model Parameters

A summary of all the parameters included in this model are given below according to each model component:

1. Main rotor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS.HUB</td>
<td>Fuselage station of hub</td>
</tr>
<tr>
<td>WL.HUB</td>
<td>Water line location of hub</td>
</tr>
<tr>
<td>IS</td>
<td>Forward tilt of rotor shaft w.r.t. fuselage</td>
</tr>
<tr>
<td>GAM.OM.16</td>
<td>Lock number * omega / 16</td>
</tr>
<tr>
<td>R.MR</td>
<td>Radius of main rotor</td>
</tr>
<tr>
<td>RPM.MR</td>
<td>RPM of main rotor</td>
</tr>
<tr>
<td>CDO</td>
<td>Blade profile drag coefficient</td>
</tr>
<tr>
<td>DM.DA1</td>
<td>Incremental rotor stiffness factor (Hinge offset and spring stiffness of rotor)</td>
</tr>
<tr>
<td>A</td>
<td>Blade lift curve slope</td>
</tr>
<tr>
<td>B</td>
<td>Number of blades</td>
</tr>
<tr>
<td>C</td>
<td>Blade chord</td>
</tr>
</tbody>
</table>

2. Fuselage

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS.FUS</td>
<td>Fuselage station of fuselage center of pressure</td>
</tr>
<tr>
<td>WL.FUS</td>
<td>Waterline station of fuselage center of pressure</td>
</tr>
<tr>
<td>XUU.FUS</td>
<td>Aerodynamic quadratic model constant</td>
</tr>
<tr>
<td>YUU.FUS</td>
<td>&quot;          &quot;       &quot;       &quot;</td>
</tr>
<tr>
<td>ZUU.FUS</td>
<td>&quot;          &quot;       &quot;       &quot;</td>
</tr>
</tbody>
</table>

3. Tail rotor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS.TR</td>
<td>Fuselage station of tail rotor</td>
</tr>
<tr>
<td>WL.TR</td>
<td>Waterline station of tail rotor</td>
</tr>
<tr>
<td>R.TR</td>
<td>Radius of tail rotor</td>
</tr>
<tr>
<td>RPM.TR</td>
<td>RPM of tail rotor</td>
</tr>
<tr>
<td>A</td>
<td>Blade lift curve slope</td>
</tr>
<tr>
<td>B</td>
<td>Number of blades</td>
</tr>
<tr>
<td>C</td>
<td>Blade chord</td>
</tr>
</tbody>
</table>

4. Horizontal tail

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS.HT</td>
<td>Fuselage station of horizontal tail</td>
</tr>
<tr>
<td>WL.HT</td>
<td>Waterline station of horizontal tail</td>
</tr>
<tr>
<td>ZUU.HT</td>
<td>Quadratic max lift coeff of horizontal tail</td>
</tr>
<tr>
<td>ZUW.HT</td>
<td>&quot;          &quot;       &quot;       &quot;</td>
</tr>
<tr>
<td>ZMAX.HT</td>
<td>&quot;          &quot;       &quot;       &quot;</td>
</tr>
</tbody>
</table>

5. Wing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS.WN</td>
<td>Fuselage station of wing</td>
</tr>
<tr>
<td>WL.WN</td>
<td>Waterline station of wing</td>
</tr>
<tr>
<td>ZUU.WN</td>
<td>Quadratic max lift coeff of wing</td>
</tr>
<tr>
<td>ZUW.WN</td>
<td>&quot;          &quot;       &quot;       &quot;</td>
</tr>
<tr>
<td>ZMAX.WN</td>
<td>&quot;          &quot;       &quot;       &quot;</td>
</tr>
</tbody>
</table>
6. Vertical tail

FS.VT           Fuselage station of vertical tail
WL.VT           Waterline station of vertical tail
YUU.VT
YUV.VT
YMAX.VT         Quadratic max lift coeff of vertical tail
III. Model Matching and Estimation Procedures

In order to demonstrate model matching and estimation procedures, a model of the Bell AH-1S Cobra is developed. The actual code for this example version along with a list of symbols and a table of associated input parameters are presented in Appendices A, B, and C.

The primary sources which are used in this example are the flight manual (Reference 11), a manufacturer's stability and control package (Reference 12), a volume of Jane's (Reference 13), and a flight dynamics data report (Reference 14). Other useful references include the USAF Stability and Control Datcom (Reference 15), the U. S. Army Engineering Design Handbook (Reference 16) and the previously cited Stepniewski and Keyes reference.

In this section, we will describe the individual components of the AH-1S and how each of the associated parameters were determined. There are 44 total parameters needed for this model. 22 of these are simple geometrical variables which can be easily obtained from a scale drawings, from aircraft manuals, or even estimated from a picture of the aircraft.

A. Mass, Loading, and Geometry Data

A substantial portion of the data required are either directly obtainable geometric data or are common mass and mass-loading data.

1. Geometric Data

Geometric parameters are easily obtained from aircraft drawings or reference literature. Figure 4, taken from the flight manual, provides a basis for geometric information. Note that positions of all major components are given relative to the manufacturer's reference system (fuselage stations, waterlines, and buttocks).

Explicit positions can be obtained for some features such as main rotor hub position and tail rotor hub. For airfoils it is generally sufficient to estimate and use the positions for one-quarter mean aerodynamic chord. The fuselage aerodynamic center is less clearly defined and must be estimated depending upon the shape. Appendages such as tail boom and landing gear can be considered in estimating the fuselage aerodynamic center.

-27-
Figure 6-1. Helicopter Station Diagram

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>FS</th>
<th>WL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main rotor hub</td>
<td>200</td>
<td>153</td>
</tr>
<tr>
<td>Tail rotor hub</td>
<td>521</td>
<td>119</td>
</tr>
<tr>
<td>Fuselage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal tail</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical tail</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Basis for Geometric Data.
2. Mass and Loading Data

Values for normal operating gross weight and center of gravity are typically obtained from operating manuals. An example is shown in Figure 5. Specific choices will depend upon the general loading condition of interest. Here an intermediate loading is chosen which also corresponds to other available data.

Inertial data from the Reference 12 stability and control report are given in Table 3. While these do not correspond exactly to the loading chosen above, they can be easily rescaled by assuming a constant radius of gyration in each axis.
EXAMPLE

WANTED
DETERMINE APPROXIMATE CENTER
OF GRAVITY
FOR KNOWN WEIGHT AND MOMENT
KNOWN
GROSS WEIGHT = 8000 POUNDS
MOMENT/100 = 18190 INCH-POUNDS
METHOD
MOVE RIGHT FROM BEAK HOURS
TO APPROXIMATE MIDPOINT
BETWEEN 18100 AND 18200 IN-LS
DIAGONAL LINES
FROM THIS POINT MOVE DOWN
TO READ 197.4 ON CENTER OF
GRAVITY SCALE

Figure 5. Basis for Loading Data.
Table 3. Basis for Inertial Data

<table>
<thead>
<tr>
<th>Condition</th>
<th>Weight (lbs.)</th>
<th>( \bar{x} ) (in.)</th>
<th>( \bar{y} ) (in.)</th>
<th>Moment of Inertia (G-in-ft)</th>
<th>Principal Axis (Mean Down Angle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Weight Empty</td>
<td>5571.4</td>
<td>204</td>
<td>82</td>
<td>1990.4</td>
<td>10592.7 8878.2</td>
</tr>
<tr>
<td>(2) Basic</td>
<td>8673.2</td>
<td>193</td>
<td>71</td>
<td>2843.0</td>
<td>13115.6 11264.0 6° 30'</td>
</tr>
<tr>
<td>(3) Hog</td>
<td>9501.1</td>
<td>194</td>
<td>68</td>
<td>4002.3</td>
<td>13082.3 11930.4 6° 37'</td>
</tr>
<tr>
<td>(4) Scout</td>
<td>9296.9</td>
<td>194</td>
<td>70</td>
<td>5195.3</td>
<td>13233.1 11656.7 6° 25'</td>
</tr>
<tr>
<td>(5) Most Forward</td>
<td>6606.2</td>
<td>191</td>
<td>78</td>
<td>2255.5</td>
<td>12462.4 10499.5 7° 19'</td>
</tr>
<tr>
<td>(6) Most Aft</td>
<td>7476.8</td>
<td>200</td>
<td>75</td>
<td>2265.6</td>
<td>11801.0 9903.8 4° 18'</td>
</tr>
</tbody>
</table>
B. Propulsion Data

Required propulsion data include power available for given operating conditions. These data can be found in Jane's under the appropriate propulsion system manufacturer as illustrated in Table 4. The specific information of interest here is the maximum continuous power rating for the AVCO Lycoming T53-L-703 gas turbine engine.

Other information needed consists of an approximate breakdown of power, including that due to accessories. Data from the Stepniewski and Keyes source are given in Table 5. These data will be used to estimate power losses from the computed power required by each of the components listed previously.

The basis for torque (power) available under various operating conditions is given in Figure 6. (Percent torque is assumed equal to percent power for the normal operating rpm--324 in this case.)
Table 4. Basis for Propulsion System Data.

AVCO LYCOMING GAS TURBINE ENGINES

<table>
<thead>
<tr>
<th>Manufacturer's and civil designation</th>
<th>Military designation</th>
<th>Type *</th>
<th>T-O Rating</th>
<th>SFC</th>
<th>Weight dry</th>
<th>Mensa</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>kN (lb st)</td>
<td>mpg/g.</td>
<td>less tailpipe</td>
<td>dia (in)</td>
<td>overall</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>or max kW (hp)</td>
<td>(lb/hp)</td>
<td>kg (lb)</td>
<td>mm (in)</td>
<td>mm (in)</td>
</tr>
<tr>
<td>T5313B</td>
<td>ACFS</td>
<td>1,044</td>
<td>98 (0-58)</td>
<td>245 (540)</td>
<td>584 (23)</td>
<td>1,209 (47-6)</td>
<td></td>
</tr>
<tr>
<td>T5313A</td>
<td>ACFS</td>
<td>1,119</td>
<td>99.7 (0-59)</td>
<td>236 (564)</td>
<td>584 (23)</td>
<td>1,209 (47-6)</td>
<td></td>
</tr>
<tr>
<td>T5311A</td>
<td>ACFS</td>
<td>820</td>
<td>115 (0-68)</td>
<td>225 (496)</td>
<td>584 (23)</td>
<td>1,209 (47-6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACFS</td>
<td>1,044</td>
<td>98 (0-58)</td>
<td>245 (540)</td>
<td>584 (23)</td>
<td>1,209 (47-6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T53-1-702</td>
<td>ACFS</td>
<td>1,106</td>
<td>101.4 (0-60)</td>
<td>247 (545)</td>
<td>584 (23)</td>
<td>1,209 (47-6)</td>
</tr>
<tr>
<td></td>
<td>ACFS</td>
<td>1,157</td>
<td>98.7 (0-58)</td>
<td>234 (515)</td>
<td>584 (23)</td>
<td>1,209 (47-6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACFS</td>
<td>1,187</td>
<td>102.7 (0-60)</td>
<td>363 (799)</td>
<td>615 (24-2)</td>
<td>1,181 (44)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACFS</td>
<td>2,125</td>
<td>101.4 (0-60)</td>
<td>267 (590)</td>
<td>615 (24-2)</td>
<td>1,181 (44)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACFS</td>
<td>2,186</td>
<td>100-1 (0-592)</td>
<td>274 (605)</td>
<td>610 (24)</td>
<td>1,181 (44)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ACFS</td>
<td>2,796</td>
<td>89.6 (0-53)</td>
<td>322 (710)</td>
<td>615 (24-2)</td>
<td>1,181 (46-5)</td>
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<td></td>
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<td>3,430</td>
<td>86.2 (0-51)</td>
<td>329 (725)</td>
<td>615 (24-2)</td>
<td>1,181 (44)</td>
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<td></td>
<td>ACFS</td>
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<td>110-19 (10-36)</td>
<td>156 (343)</td>
<td>564 (33)</td>
<td>890 (33)</td>
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<td>111-64 (10-411)</td>
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<td>1,059 (41-7)</td>
<td>1,183 (46-8)</td>
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<tr>
<td></td>
<td>ACFS</td>
<td>11.4</td>
<td>122-1 (10-428)</td>
<td>590 (1,208)</td>
<td>1,059 (41-7)</td>
<td>1,183 (58-36)</td>
<td></td>
</tr>
</tbody>
</table>

*ACFS = axial plus centrifugal, free-turbine shaft; ACFF = axial plus centrifugal, free-turbine propeller; ACFF = axial plus centrifugal, free-turbine fan
†Applies to T53-11A, C **, D, E ** and 712 **, those designated ** having 2½ min contingency rating of 3,357 kW (4,500 shp)

Table 5. Assumed Breakdown of Power Absorption.

<table>
<thead>
<tr>
<th></th>
<th>% total power in hover</th>
<th>% total power max forward</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main rotor induced power</td>
<td>65</td>
<td>15</td>
</tr>
<tr>
<td>Main rotor profile power</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>Fuselage parasite power</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Tail rotor total power</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Misc. and accessories</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

(Note: Power losses due to wing stall should also be considered where the effect is suspected to be significant. It will be neglected in this example.)
TORQUE AVAILABLE (CONTINUOUS OPERATION)

ENGINE DEICE OFF, ECS OFF
100% RPM JP-4 FUEL

EXAMPLE

WANTED
INDICATED TORQUE
CALIBRATED TORQUE

KNOWN
PRESSURE ALTITUDE = 7000 FEET
SAT = X; 10°C
CALIBRATION FACTOR = 84

METHOD
ENTER PRESSURE ALTITUDE HERE
MOVE RIGHT TO SAT
MOVE DOWN TO CALIBRATION FACTOR
MOVE LEFT, READ INDICATED
TORQUE = 88.0 % 0

FOR CALIBRATED TORQUE
CONTINUE DOWN THRU
CALIBRATION FACTOR, READ
CALIBRATED TORQUE = 84.2 %

DATA BASIS: CALCULATED

Figure 7-4. Torque available (continuous operation) chart (Sheet 1 of 2)

7-12

Figure 6. Basis for Torque (Power) Limits.

For sea level, std day; torque is limited by max continuous operating condition. (88%)
C. Rotor Data

Rotor system characteristics consist of geometric, aerodynamic, and operating condition features. Most of the geometric data including size and number of blades and hub center are easily found in flight manuals. Operating conditions, namely the normal operating rpm, are likewise obtained.

The main aerodynamic parameters include the effective section lift curve slope and profile drag coefficient. Commonly accepted values of 5.7 and .006, respectively, are sufficient starting points.

The most crucial rotor parameters, however, are those relating to the effective flapping stiffness or hinge offset. These data are generally found only in manufacturers design reports. Of course in the case of a simple teetering rotor the effective hinge offset is zero. Articulated rotor designs are also fairly easy to represent as long as the geometric hinge offset is known. The most difficult variety to model is the hingeless rotor since both an effective hinge offset and flapping spring must be determined.

Useful auxiliary information for modeling the rotor system is response data which provides direct indication of the unaugmented pitch and roll damping.

D. Aerodynamic Features

Aside from the rotor system aerodynamics, parameters must be estimated for the airfoil and fuselage components. The techniques for doing so are common and require little effort. If manufacturer's stability and control data are available these calculations are trivial. Otherwise, one can refer to estimation handbooks such as the USAF DATCOM (Reference 18).

Airfoil lift parameters involve three main features: camber and incidence, circulation lift, and stall. The first two are highly dependent upon geometry and the third on maximum lifting performance.

Relationships which are needed for setting parameters involve the quadratic aerodynamic parameters and the more common non-dimensional aerodynamic coefficients. These are given below for use in the estimation procedures described in Figure 7. The equations are for the horizontal tail, but the other airfoil surfaces are similar.

Estimates typical for airfoils:
\[ z_{uu} = -s h t \ c_{L_{o}} \]  \( C_{o} \) is set by both camber and incidence of the airfoil.

\[ z_{uu} = -s h t \ c_{L_{x}} \]  note that \( C_{\alpha} \approx \frac{2 \pi \ v \ R}{R+2} \)

\[ z_{\text{min}} = s h t \ c_{L_{\max}} \]  typical values are 1.5 to 3 depending upon \( R \).

Similarly, fuselage drag estimates can be made for each of the three axes using available drag data.

Estimates typical for fuselage drag:

\[ X_{uu}^{\text{fus}} = -S^{\text{fus}} C_{D} \]

where \( S^{\text{fus}} \) is projected frontal area

\( C_{D} \) can be estimated using numerous textbook tabulations of 3-d drag. This will vary for each axis.
WING:
span = 10.75
chord = 3.0
area = 32.25
AR = 3

\[ CL_e = \frac{2\pi R}{R^2 + L^2} = 5 \]

(based on \( \theta = 19^\circ \))

HORIZONTAL TAIL:
span = 7
chord = 2.25
area = 16
AR = 3

\[ CM = 1.2 \quad \text{Assume } Cl_{max} = 2 \]

VERTICAL TAIL:
span = 5
chord = 3.3
area = 11
AR = 1.5

\[ CM = 2.7 \quad \text{Assume } Cl_{max} = 3 \]

FUSELAGE:
Assumed drag coefficients:
front \( D \)
side \( D \)
up \( D \)

Figure 2-2. Principal Dimensions (Typical)

Figure 7. Basis for Initial Estimates of Aerodynamic Parameters.
E. Hover Performance

The parameters listed above provide a starting point for the math model. Additional flight manual and available flight data will serve to make refinements in model response and performance characteristics.

The first adjustment of model parameters can be made based on the flight manual hover performance as shown in Figure 8. Here the percent maximum torque is given for a specific hover condition.

The factors which can be adjusted to achieve a good match are the power losses due to accessories, downwash on the fuselage and horizontal airfoils, or main rotor induced velocity factor (if included).
Figure 8. Basis for Hover Power Required.
F. Forward Flight Data

Up to this point model adjustments have centered on the main rotor system since body drag has been low due to the hover condition. With the consideration of forward flight the fuselage now plays a major role in limiting maximum speed and climb performance.

The main set of data useful for adjusting fuselage drag are given in Figure 9 from the flight manual. Note that the primary information is the torque required as a function of flight condition and loading. The two main features on this plot are the maximum speed at continuous operating torque and the torque and speed for level flight at minimum power.

Additional information is given in Figure 10 with the maximum rate of climb corresponding to an increase in torque.

Finally in Figure 11 data are given for the maximum glide and minimum rate of descent. These are useful for setting the effective full-down collective pitch stop.
Figure 9. Basis for Forward Flight Speeds and Power Required.
Figure 10. Basis for Max Rate of Climb Speed and Power Required.
Figure 11. Basis for Max Glide Speed and Descent Angle.
As a final note, the process of tuning model parameters should not be done without careful consideration of all secondary effects. The best policy is to avoid making anything other than simple direct first-principles corrections. There is substantial redundancy in some of the data shown here and it is not possible to achieve perfect matches in all respects. One needs to exercise judgement in where a reasonable match has been attained and should be accepted as adequate.
IV. Checkout Procedures

A. General

As discussed earlier, model complexity can hamper the thoroughness of simulator computer program implementation and checking. However, the model presented here can be fully checked with reasonable effort. This is due to the fewnness of model constants, fewnness of degrees of freedom, and minimal program branching. The recommended checking procedure involves the following elements:

- Use of an independent operating program.
- Verification of trim points.
- Verification of state transitions through n steps.
- Overlay of time histories.
- Identification of dominant response modes.

Some of these steps are redundant but nevertheless serve to build confidence in the correctness of the math model implementation at only minimal added cost. The following is a brief discussion of each element.

B. Discussion of Checkout Procedure Elements

1. Independent Operating Program

As a general rule, math model checkout should be accomplished using an independent implementation and check source. Furthermore, not only should an independent program be used but also an independent computer.

This math model form enables the user to develop a math model version on a small desktop microcomputer and run complete sets of check cases well in advance of using the simulator computer facilities.

The specific computer system used to develop and run this math model consisted of a Compaq 286 desktop computer with 640K working memory running Microsoft Basic. Only an interpreter mode was used although a Basic compiler is available. The interpreter permits a highly efficient interaction between the model developer and the computer system.
2. Trim Point Verification

A check of static trim points gives an initial indication of correct model implementation. The full operating envelope can be covered with just a few cases and possible discrepancies isolated to airspeed, vertical velocity, or controls. A cursory check of suspected parameters or component equations can usually lead to simple corrections. Trim solutions should be correct prior to proceeding to the next item.

A sample of the trim solution printout is given in Figure 12. This same format is displayed during the trimming process so that one can observe whether there are difficulties in iterating on a solution.

**TRIM CALCULATIONS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{P} )</td>
<td>1.68E-01</td>
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<tr>
<td>( \dot{Q} )</td>
<td>-7.82E-03</td>
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<td>( \dot{R} )</td>
<td>4.44E-02</td>
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<td>( \dot{U} )</td>
<td>-9.73E-04</td>
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<tr>
<td>( \dot{V} )</td>
<td>-9.03E-03</td>
</tr>
<tr>
<td>( \dot{W} )</td>
<td>3.84E-03</td>
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<td>( \dot{\alpha} )</td>
<td>2.59E-02</td>
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<tr>
<td>( \dot{\beta} )</td>
<td>2.55E-01</td>
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<td>( \dot{\phi} )</td>
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<tr>
<td>( \dot{\theta} )</td>
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<tr>
<td>( \dot{\psi} )</td>
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<tr>
<td>( \dot{\chi} )</td>
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<tr>
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<td>( \dot{\psi} )</td>
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<table>
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<th>D.W.?</th>
<th>Stall condition.</th>
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<tr>
<td>1</td>
<td>Hor. tail O.K.</td>
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<tr>
<td>0</td>
<td>Wing O.K.</td>
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**Figure 12. Sample of Trim Point Printout.**
3. State Transition Verification

Given that static solutions are valid, the dynamic response characteristics should be examined next. Correct operation is indicated by tracking several discrete state variable transitions and comparing with independently obtained check values. This is made feasible by restricting the number of degrees of freedom and levels of numerical integration. For example, only about six transitions for each control variable are needed to excite each term in the model equations.

In order to thoroughly check state transitions, a table overlay is recommended. This is accomplished by duplicating the state transition printout format of the checkout computer with that of the simulator computer. The original checks can be printed on transparencies then directly overlaid with the simulator printout.

Examples of the state transition checks are given in Table 6.
<table>
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<th>Time</th>
<th>DC(1)</th>
<th>DC(2)</th>
<th>DC(3)</th>
<th>DC(4)</th>
<th>VA(1)</th>
<th>VA(2)</th>
<th>VA(3)</th>
<th>VA(4)</th>
<th>VA(5)</th>
<th>VA(6)</th>
<th>VB(1)</th>
<th>VB(2)</th>
<th>VB(3)</th>
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<td>0.000</td>
<td>-5.545</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>101.17</td>
<td>0.000</td>
<td>-5.545</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>3.9164</td>
<td>101.17</td>
<td>-1.000</td>
<td>-5.551</td>
<td>0.003</td>
<td>-0.117</td>
<td>0.000</td>
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</tr>
</tbody>
</table>

**Configuration:** 101  
**Helicopter:** AH-1S  
**Weight:** 9000  
**ST:** .01  
**Control KC(2) Step Input:** 5 DEGREES  
**Note:** All angle units are degrees
4. Time History Overlays

In theory the combination of static and state transition checks should be sufficient to demonstrate agreement with the independent model implementation. However, additional confidence is gained by selecting several time history cases to overlay. These can be supplemented by checking dominant response modes based on transfer function solutions from the original independent check model.

Useful time histories to consider are angular rates for both on- and off-axes for a given control input. This checks both the dominant response modes and the amount of off-axis cross coupling. Examples are shown in Figure 13 corresponding to the previous check information.
Figure 13. Examples of Time Histories to be Used for Overlays.
5. Dominant Response Identification

It is also useful to supplement the above checks with a comparison of identified dominant response features from the simulator computer with those features observed or computed from the independent checkout version. This is particularly important for handling qualities investigations.

Dominant modes are examined by exciting an axis with the corresponding direct control and scaling the appropriate first- or second-order response features from the respective motion traces. The on-axis traces presented earlier in Figure 13 serve this purpose for extracting short-term pitch repose information.
V. Model Extensions and Refinements

The example which has been presented above can be modified in a number of ways in order to address specific simulation needs. The above math model can be either simplified or made more sophisticated. The following is a discussion of some possible extensions and refinements.

A. Flight Control System

There is no flight control system included in the above model other than conventional aerodynamic interfaces such as cyclic, collective, and tail rotor controls. Addition of a flight control system requires definition of relationships between the cockpit manipulator and the above aerodynamic controls plus any stability and control augmentation systems.

As with the basic airframe math model, definition of flight controls can be done with a wide range of computational complexity. However, the same considerations can be applied in order to match the level of complexity with user utility. The main question is to what degree can the simulator pilot observe or be influenced by math model intricacies.

B. Engine-Governor

This aspect of the helicopter math model can be important for tasks involving maneuvering or aggressive control of collective pitch.

The above math model is designed to accommodate an engine-governor system since rotor speed is explicit in the equations. It is necessary only to add appropriate engine governor equations of motion prior to computation of the main rotor thrust.

In general, only a second-order engine governor response is required in order to handle the effective spring-mass-damper action of the main rotor combined with the propulsion system and governor control laws. An adequate model is described in Reference 17.

C. Ground Effect

The modeling of ground effect can be important for tasks involving hover under marginal performance conditions. Again, the computational complexity of such models can vary widely.

It is recommended that, as a first cut, ground effect be modeled as an induced-velocity efficiency factor which primarily affects the power-required to hover. This efficiency factor can be adequately modeled as an exponential function of altitude. The exponential scale height and magnitude is easily quantified from the flight manual hover performance shown earlier in Figure 8.
D. Induced Flow Dynamics

For certain vertical response applications it may be important to model the effective lag in thrust due to a collective pitch change. This is typically a first-order lag in the range of 10 to 15 rad/sec and varies with the sign of the collective pitch change.

This effect can be modeled by setting a first-order lag on the calculation of thrust and induced velocity. Reference 18 can be consulted for guidance in setting values.

E. Higher-Order Flapping, Coning, and Lagging

Higher order rotor system dynamics may be of interest when examining flight control system schemes or certain vibrational effects. However the modes can easily be outside the computational ability of the simulator or highly distorted by the motion system. Thus is crucial for the modeler to analyze computational requirements relative to capabilities.
REFERENCES


APPENDIX A

BASIC PROGRAM LISTING OF MATH MODEL

5930 '*************************************************
5940 'DYNAMICS: Dynamic subroutine
5950 '*************************************************
5960 'A/C relative to air mass
5970 'C4 = COS(XE(4)) : S4 = SIN(XE(4))
5980 'C5 = COS(XE(5)) : S5 = SIN(XE(5))
5990 'C6 = COS(XE(6)) : S6 = SIN(XE(6))
6000 'VA(1)=VR(1)-(-VG(1)*C5)
6010 'VA(2)=VB(2)-(VG(2)*C6-VG(1)*S6)
6020 'VA(3)=VB(3)-(VG(3)*C5+VG(1)*S5)
6030 'VA(4)=VB(4)-VG(4)
6040 'VA(5)=VB(5)
6050 'VA(6)=VB(6)

6060 'Quadratic States (relative air mass velocities)
6070 UU=VA(1)+VA(1) : UV=VA(1)+VA(2) : UW=VA(1)+VA(3)
6080 VV=VA(2)+VA(2) : VW=VA(2)+VA(3) : WW=VA(2)+VA(3)
6090 UP=VA(1)+VA(4) : UG=VA(1)+VA(5) : UR=VA(1)+VA(6) : MR=VA(3)+VA(6)
6100 VTA=SQR(UU+VV+WW)
6110 IF VA(1)<3! THEN ALPHA.F= XE(5) : GOTO 6210
6120 ALPHA.F=ATN(VA(3)/VA(1))

6130 'Integrate tip path plane angles
6140 SV(7)=SV(7) + ST*(A2+F7)*GAM.OH.16 + B2*AP(7)
6150 SV(8)=SV(8) + ST*(A2+F8)*GAM.OH.16 + B2*AP(8)
6160 AF(7)=F7*GAM.OH.16 : AF(8)=F8*GAM.OH.16

6170 '************ Main Rotor thrust and induced velocity ************
6180 Rotor thrust calculation
6190 'Compute z-axis velocity relative to rotor plane (Mr) and blade (Mb):
6200 MR = VA(3) + (SV(7) + IS)*VA(1) - SV(8)*VA(2)
6210 MB = MR + 2/3*OMEGA.MR*MR*(DC(1) + .75*THETA.TWIST)

6220 'Perform iterative solution of thrust and induced velocity
6230 FOR I=1 TO 5
6240 THRUST.MR=(MB-MR)*OMEGA.MR*MR*MR*MR/4
IF THRUST.MR>0 THEN THRUST.MR=0!

VHAT.2=VA(1)*2 + VA(2)*2 + MR(MR-2*VI.MR)

VI.MR.2=SQRT(VHAT.2/2)*(VHAT.2/2)+(THRUST.MR/2)/(OMEGA.MR*MR)*2)/ - WHAT.2/2

VI.MR=SQRT(ABS(VI.MR.MR.2))

IF VI.MR.2<0 THEN VI.MR=-VI.MR

NEXT I

* Compute DAIDU and DBIDV

DBIDV = (B3*D/C1/((OMEGA.MR+MR.MR) + 2*(VA(3)-VI.MR)/(OMEGA.MR+MR.MR)^2

DAIDU = DBIDV*(1 + (3/2)*VA(1)^2/(OMEGA.MR+MR.MR)^2)

'**************************** Fuselage ***************************

WA.FUS = VA(3) - VI.MR

X.FUS = SDN(VA(1))*R2*XUJJ.FUS+UW

Y.FUS = SDN(VA(2))*R2*YYJ.FUS+VY

Z.FUS = SDN(WA.FUS)*R2*WWJ.FUS+WA.FUS+2

L.FUS = Y.FUS*H.FUS

M.FUS = Z.FUS*H.FUS - X.FUS*H.FUS

N.FUS = -Y.FUS*H.FUS

'**************************** Main rotor power and torque ****************************

P.INDUCED.MR = THRUST.MR+VI.MR+KIND

P.CLIMB = MT4HDOT

P.HYDRAULIC = ABS(X.FUS*VA(1)) + ABS(Y.FUS*VA(2)) + ABS(Z.FUS*WA.FUS)

POWER.MR = P.INDUCED.MR + P.CLIMB + P.HYDRAULIC + P.PROFILE.MR

PENDS.MR = P.INDUCED.MR + P.PROFILE.MR

POWER.FUS = P.HYDRAULIC

TORQUE.MR = POWER.MR/OMEGA.MR

' Compute main rotor force and moment components.

I.MR = -THRUST.MR*SIN(GV(7))

Y.MR = THRUST.MR*SIN(GV(8))

Z.MR = -THRUST.MR*COS(GV(7))*COS(GV(8))

Add DM.DA1 moment contribution to L and M equations.

L.MR = Y.MR*H.HUB + DM.DA1*GV(8)


N.MR = -Y.MR*H.HUB + TORQUE.MR

'**************************** Tail Rotor thrust and induced velocity ****************************

Rrotor thrust calculation

Compute y-axis velocity relative to rotor plane (Mr) and blade (Mb):
YRTR = -(VA(2) - VA(6)*D.TR + VA(4)*H.TR)
YBTR = YRTR +2/3*OMEGA.TR*R.TR*10C(4) + .75*THETA.TWIST.TR)

Perform iterative solution of thrust and induced velocity

FOR I=1 TO 20
   THRUST.TR=(YBTR-VI.TR)*OMEGA.TR*R.TR*R.HD*THR.isDebugEnabled.TR(R.TR/4)
   IF THRUST.TR=0 THEN THRUST.TR=0
   WHAT.2=(VA(3)+VA(5)*D.TR)**2 + VA(1)**2 + YMTR#(YBTR-2*VI.TR)
   VI.TR.2=SGR((WHAT.2/2)*(WHAT.2/2)+(THRUST.TR/7)(R.HD*THR.isDebugEnabled.TR*71))**2 - WHAT.2)
   VI.TR=SGR(AVS(VI.TR.2))
   IF VI.TR.2<0 THEN VI.TR=-VI.TR
   NEXT I

************** Tail rotor power and torque **************

P.INDUSTR.TR=THRUST.TR+VI.TR*OMEGA.TR
P.PROFILE.TR = R2*(FR.TR/4)*OMEGA.TR.R.TR*2*4.6**(IU+(VA(3)+VA(5)*D.TR)**2))
POWER.TR = P.INDUSTR.TR + P.PROFILE.TR
R.INDUSTR.TR=POWER.TR/OMEGA.TR

Compute tail rotor force and moment components.

Y.TR = THRUST.TR
L.TR = Y.TR+H.TR
M.TR = -Y.TR+H.TR
N.TR = -Y.TR+H.TR

************** Horizontal tail **************

Check if horizontal tail is in main rotor downwash

Compute aerodynamic force on tail

IF VA(1)<2! THEN EPSILON_HT=1: GOTO 7330
THETA.IND=ATN(VI.MR/VA(1))
IF THETA.IND<=THETA.CR1HT THEN EPSILON_HT=1! ELSE EPSILON_HT=0
WA.HT = VA(3) - EPSILON_HT*VI.MR+D.HT*VA(5)
I.HT=R2*(IU1+HT*HT+IU1*HT+VA(1)*WA.HT)

Check if horizontal tail is stalled

IF Z.HT < R2*MAXHT*NU THEN Z.HT=R2*MAXHT*NU ELSE GOTO 7400
IF STALLOFF=0 THEN LOCATE 21,55 : PRINT "Horizontal tail stall!" ; LOCATE 21,46 :PRINT EPSILON_HT: GOTO 7410
IF STALLOFF=0 THEN LOCATE 21,55 : PRINT "Hor. tail D.K. " ; LOCATE 21,46 :PRINT EPSILON_HT

Compute horizontal tail moments

MW.HT = Z.HT D.HT

************** Wing **************
7480 ' Check if wing is in main rotor downwash
7490 ' Compute aerodynamic force on wing
7500 ' IF VA(1)<2 THEN EPSILON.WN=1 : GOTO 7540
7520 IF THETA.IND = > THETA.CRIT.WN THEN EPSILON.WN=1 ELSE EPSILON.WN=0
7530 ' I.WN=R2*(ZUW.WN+UU + ZUW.WN*VA(1)/((VA(1)-EPSILON.WN*VI.MR)))
7550 I.WN=R2*(1/(PI#B.WN^2))*(ZUW.WN+2*UU+2*ZUW.WN*ZUW.WN+VA(1)^2-RAIL.WN*VI.MR)
7560 ' IF I.WN < R2+2*MAX.WN=UU THEN Z.WN=R2+2*MAX.WN=UU ELSE GOTO 7610
7580 IF STALLOFF=0 THEN LOCATE 22,55 : PRINT "Wing stall!": LOCATE 22,46:PRINT EPSILON.WN: GOTO 7620
7600 IF STALLOFF=0 THEN LOCATE 22,55 : PRINT "Wing O.K.": LOCATE 22,46:PRINT EPSILON.WN
7620 ' Compute wing moments
7640 ' M.WN = I.WN+D.WN - I.WN+H.WN
7660 ' Compute power into wing induced drag
7680 ' POWER.WN = ABS(I.WN+VA(1))
7700 ' POWER = POWER.MR + POWER.TR + POWER.WN + HP.LOSS*S50
7720 ' ************************ Vertical tail *************************
7740 ' Compute aerodynamic forces on vertical tail
7750 ' Y.VT=R2*(YUW.VT+YUV.VT+VA(1)*(VA(2)-VA(1)+D.VT))
7770 ' Check if vertical tail is stalled
7790 ' IF Y.VT < R2+2*MAX.VT=UU THEN Y.VT=R2+2*MAX.VT=UU ELSE GOTO 7830
7820 IF STALLOFF=0 THEN LOCATE 20,55 : PRINT "Vertical tail stall!": LOCATE 20,47 : PRINT "": LOCATE 13,1 : GOTO 7840
7840 ' IF STALLOFF=0 THEN LOCATE 20,55 : PRINT "Vert. tail O.K.": LOCATE 20,47 : PRINT "": LOCATE 13,1
7860 ' Compute vertical tail moments
7870 ' Y.VT = Y.VT-D.VT
7880 ' Compute vertical tail moments
7900 ' ************************ General force equations *************************
7910 ' I.BRAV = -M.BRAV*S5
7930 Y.BRAV = M.BRAV*5*S4+C5
7940 Z.BRAV = M.BRAV*5*C+S4+C4
7990 '1          1          1          1          1          1          1
B000 F(1) = X.GRAV + X.MR + X.FUS + X.WN  
B010 F(2) = Y.GRAV + Y.MR + Y.FUS + Y.TR + Y.VT  
B020 F(3) = Z.GRAV + Z.MR + Z.FUS + Z.IHT + Z.WN  
B030 F(4) = + L.MR + L.FUS + L.TR + L.VT  
B040 F(5) = + M.MR + M.FUS + M.TR + M.IHT + M.WN  
B050 F(6) = + N.MR + N.FUS + N.IHT + N.VI  
B060 :  
B070 :  
B080 :  
B090 F(7)=DC(3)-VA(15)/SAM.DM.16 - 6V(7) + DAID~VA(1)  
B100 F(8)=DC(2)-VA(14)/SAM.DM.16 - 6V(8) - DBID~VA(2)  
B110 '  
B120 '  
B130 '  
B140       60SUB 10900  
B150 '  
B160 '  Body Accelerations  
B170 '  
B180 AB(1) = (VB(5)*VB(3)-VB(6)+VB(2)) + F(1)/M  
B190 AB(2) = (VB(4)*VB(3)-VB(1)+VB(6)) + F(2)/M  
B200 AB(3) = (VB(1)*VB(5)-VB(4)+VB(2)) + F(3)/M  
B210 AB(4) = F(4)/Y  
B220 AB(5) = F(5)/Y  
B230 AB(6) = F(6)/IZ  
B240 '  
B250 '  Integrate Body Accelerations  
B260 '  
B270 '  FOR IX = 1 TO 6  
B280 VB(IX) = VB(IX) + ST + (AI * AB(IX) + BI * AP(IX))  
B290 AP(IX) = AB(IX) : REM SAVE ACCEL PAST VALUES  
B300 NEXT IX  
B310 '  
B320 '  Transform to earth (A/C rel to deck) velocities  
B330 '  
B340 VE(1) = (VB(1) * CS + VB(3) * SS) * C4 + C05 * (XE(6))  
B350 VE(2) = VB(2) * COS(XE(6)) + VB(1) * SIN(XE(6))  
B360 VE(3) = (VB(1) * SS - VB(3) * CS) * C4  
B370 VE(4) = VB(4) + (VB(5) * SS + VB(6) * C4) * TAN(XE(5))  
B380 VE(5) = VB(5) * C4 - VB(6) * SS  
B390 VE(6) = (VB(6) * C4 + VB(5) * SS) / CS  
B400 '  
B410 '  Integrate earth (A/C relative to deck) velocities  
B420 '  
B430 '  FOR IX = 1 TO 6  
B440 XE(IX) = XE(IX) + ST + (AI * VE(IX) + BI * UP(IX))  
B450 UP(IX) = VE(IX) : REM SAVE VEL PAST VALUES  
B460 NEXT IX  
B470 '  
B480 TIME=FIME+ST  
B490 '  
B500 RETURN  
B510 '  

A-5
APPENDIX B
DEFINITION OF PROGRAM SYMBOLS

A1  Numerical integration constant (Adams-two = 1.5).
A2  Numerical integration constant (trapezoidal = .5).
ALT&H.A.F  Fuselage angle of attack (rad).

AB(1)  Body x-axis acceleration (U) component (ft/sec²).
AB(2)  Body y-axis acceleration (V) component (ft/sec²).
AB(3)  Body x-axis acceleration (W) component (ft/sec²).
AB(4)  Body roll axis acceleration (P) component (rad/sec²).
AB(5)  Body pitch axis acceleration (Q) component (rad/sec²).
AB(6)  Body yaw axis acceleration (R) component (rad/sec²).
AB(7)  Lateral tip-path-plane angle (rad).
AB(8)  Longitudinal tip-path-plane angle (rad).
AP(1)  Past value of AB(1)
C4  Cos[XE(4)] or Cos of roll Euler angle.
C5  Cos[XE(5)] or Cos of pitch Euler angle.
C6  Cos[XE(6)] or Cos of yaw Euler angle.
DA1DU  Partial of longitudinal flapping to forward velocity (rad/ft/sec).
DB1DV  Partial of lateral flapping to side velocity (rad/ft/sec).
DC(1)  Main rotor collective pitch angle (rad).
DC(2)  Lateral swashplate angle
DC(3)  Longitudinal swashplate angle
DC(4)  Tail rotor collective pitch angle (rad).
D.FUS  Fuselage horizontal position of aerodynamic center (ft).
D.HT  Hub horizontal position (ft).
D.HUB  Hub horizontal position (ft).
D.TR  Tail rotor horizontal position (ft).

DM.DA1  Lumped flapping stiffness due to hinge offset and flapping spring (ft-lb/rad).

EPSILON.HT  Flag for rotor downwash on horizontal tail.

EPSILON.WN  Flag for rotor downwash on wing.

F(1)  Total x-force component (lb).
F(2)  Total y-force component (lb).
F(3)  Total z-force component (lb).
F(4)  Total rolling moment component (ft-lb).
F(5)  Total pitching moment component (ft-lb).
F(6)  Total yawing moment component (ft-lb).
F(7)
F(8)

GAM.OM.16  One-sixteenth the product of Lock No. and rotor angular rate (rad/sec).

GV(7)  Longitudinal tip-path-plane angle rate (rad/sec).
GV(8)  Lateral tip-path-plane angle rate (rad/sec).
H.FUS  Fuselage vertical position of aerodynamic center (ft).
H.HUB  Hub vertical position (ft).
H.TR  Tail rotor vertical position (ft).
HUB  Shaft incidence (rad).
KIND  Induced velocity factor.
L.FUS  Fuselage aerodynamic rolling moment (ft-lb).
L.MR  Main rotor rolling moment (ft-lb).
L.TR  Tail rotor rolling moment (ft-lb).
L.VT  Vertical tail rolling moment (ft-lb).
M  Vehicle mass (slug).
M.FUS  Fuselage aerodynamic pitching moment (ft-lb).
M.HT  Fuselage aerodynamic yawing moment (ft-lb).
M.MR  Fuselage aerodynamic rolling moment (ft-lb).
M.TR  Tail rotor pitching moment (ft-lb).
M.WN  Tail rotor rolling moment (ft-lb).
N.FUS  Fuselage aerodynamic yawing moment (ft-lb).
N.MR
N.TR. Tail rotor yawing moment (ft-lb).
N.VT
OMEGA.MR Main rotor angular rate (rad/sec).
OMEGA>TR Tail rotor angular rate (rad/sec).
P.CLIMB Power loss due to change in potential energy (ft-lb/sec).
P.INDUCED.MR Power loss due to main rotor induced velocity (ft-lb/sec).
P.INDUCED.TR Power loss due to tail rotor induced velocity (ft-lb/sec).
P.PARASITE Power loss due to fuselage parasite drag (ft-lb/sec).
P.PROFILE.MR Power loss due to main rotor profile drag (ft-lb/sec).
P.PROFILE.TR Power loss due to tail rotor profile drag (ft-lb/sec).
POWER Total power required (ft-lb/sec).
POWER.FUS Total power absorbed by fuselage, including parasite drag (ft-lb/sec).
POWER.MR
POWER.TR
POWER.WN
POWER.ROTOR.MR
R2 One half air density (slug/ft^3).
RHO.ABC.MR Main rotor product of air density, lift-slope, number of blades, and chord ( ).
RHO.ABC.TR Tail rotor product of air density, lift-slope, number of blades, and chord ( ).
R.MR Main rotor radius (ft).
R.TR Tail rotor radius (ft).
S4 Sin[XE(4)] or Sin of roll Euler angle.
S5 Sin[XE(5)] or Sin of pitch Euler angle.
S6 Sin[XE(6)] or Sin of yaw Euler angle.
ST Step size (sec).
STALLOFF Stall flag.
THETA.CRIT.HT
THETA.CRIT.WN
THETA.IND
THETA.TWIST Main rotor blade twist (rad).
THETA.TWIST.TR Tail rotor blade twist (rad).
THRUST.MR Main rotor thrust (lb).
THRUST.TR Tail rotor thrust (lb).
TIME Present time (sec).
TORQUE.MR Main rotor torque (ft-lb).
TORQUE.TR Tail rotor torque (ft-lb).

UU Quadratic airspeed product VA(1)*VA(1) (ft²/sec²).
UV Quadratic airspeed product VA(1)*VA(2) (ft²/sec²).
UW Quadratic airspeed product VA(1)*VA(3) (ft²/sec²).
UP Quadratic airspeed product VA(1)*VA(4) (rad²/sec²).
UQ Quadratic airspeed product VA(1)*VA(5) (rad²/sec²).
UR Quadratic airspeed product VA(1)*VA(6) (rad²/sec²).

VA(1) Airspeed vector along x-axis (ft/sec).
VA(2) Airspeed vector along x-axis (ft/sec).
VA(3) Airspeed vector along x-axis (ft/sec).

VA(4) Angular velocity of relative airmass about x-axis (rad/sec).
VA(5) Angular velocity of relative airmass about y-axis (rad/sec).
VA(6) Angular velocity of relative airmass about z-axis (rad/sec).

VB(1) Inertial (U) velocity vector along x-axis (ft/sec).
VB(2) Inertial (V) velocity vector along y-axis (ft/sec).
VB(3) Inertial (W) velocity vector along z-axis (ft/sec).

VB(4) Inertial angular rate (P) about x-axis (rad/sec).
VB(5) Inertial angular rate (Q) about y-axis (rad/sec).
VB(6) Inertial angular rate (R) about z-axis (rad/sec).

VE(1) Earth axis forward velocity (ft/sec).
VE(2) Earth axis sideward velocity (ft/sec).
VE(3) Vertical velocity (ft/sec).
VE(4) Roll Euler angle rate (rad/sec).
VE(5)  Pitch Euler angle rate (rad/sec).
VE(6)  Heading Euler angle rate (rad/sec).
VG(1)  Airmass (gust) velocity along x-axis (ft/sec).
VG(2)  Airmass (gust) velocity along y-axis (ft/sec).
VG(3)  Airmass (gust) velocity along z-axis (ft/sec).
VG(4)  Airmass (gust) angular rate about x-axis (rad/sec).
VG(5)  Airmass (gust) angular rate about y-axis (rad/sec).
VG(6)  Airmass (gust) angular rate about z-axis (rad/sec).
VI.MR  Main rotor induced velocity (ft/sec).
VI.MR.2 VI.MR squared.
VI.TR  Tail rotor induced velocity (ft/sec).
VP(i)  Past value of VE(i)
VU.TR.2 VI.TR squared.
WHAT.2 Intermediate variable in thrust calculations (ft/sec).
VTA   Total airspeed (ft/sec)
VV    Quadratic airspeed product VA(2)*VA(2) (ft²/sec²).
WW    Quadratic airspeed product VA(2)*VA(3) (ft²/sec²).
WA.FUS Apparent vertical velocity on fuselage (ft/sec).
WA.HT  Apparent vertical velocity on horizontal tail (ft/sec).
WB    Net vertical velocity component relative to the blade (ft/sec).
WR    Net vertical velocity component through the actuator disc (ft/sec).
WW    Quadratic airspeed product VA(3)*VA(6) (rad²/sec²).***
WW    Quadratic airspeed product VA(3)*VA(3) (ft²/sec²).
XE(1)  X-axis position (ft).
XE(2)  Y-axis position (ft).
XE(3)  Z-axis position (ft).
XE(4)  Roll Euler angle (rad).
XE(5)  Pitch Euler angle (rad).
XE(6)  Heading Euler angle (rad).
X.FUS  Fuselage aerodynamic x-force (lb).
X.GRAV X-gravity force (lb).
X.MR   Main rotor x-force (lb).
X.HT  Horizontal tail x-force (lb).
X.WN  Wing x-force (lb).
XUU.FUS  Fuselage quadratic drag coefficient along x-axis (ft^2).
Y.FUS  Fuselage aerodynamic y-force (lb).
Y.GRAV  Y-gravity force (lb).
YMIN.VT  Vertical tail stall effect (ft^2).
Y.MR  Main rotor side force (lb).
YBTR  Y-axis velocity relative to tail rotor blade (ft/sec).
YRTR  Y-axis velocity relative to tail rotor disk (ft/sec).
Y.TR  Tail rotor side force (lb).
YUU.VT  Vertical tail camber/incidence effect (ft^2).
YUW.VT  Vertical tail circulation lift effect (ft^2).
Y.VT  Vertical tail side force (lb).

YVV.FUS  Fuselage quadratic drag coefficient along y-axis (ft^2).
ZMIN.HT  Horizontal tail max aero force (stall) coefficient (ft^2).
ZMIN.WN  Wing max aero force (stall) coefficient (ft^2).
Z.FUS  Fuselage aerodynamic z-force (lb).
Z.GRAV  Z-gravity force (lb).
Z.HT  Horizontal tail z-force (lb).
Z.MR  Main rotor z-force (lb).
Z.WN  Wing z-force (lb).
ZWW.FUS  Fuselage quadratic drag coefficient along z-axis (ft^2).
ZUU.HT  Horizontal tail camber/incidence effect (ft^2).
ZUU.WN  Wing camber/incidence effect (ft^2).
ZUW.HT  Horizontal tail circulation effect (ft^2).
ZUW.WN  Wing circulation effect (ft^2).
APPENDIX C
DEFINITION OF PROGRAM INPUT AND DATA FILES

Two data sets are defined, first the input data file needed for a specific vehicle and second the computed data file which defines trim and dynamic data for a given flight condition case.

The following describes the input format needed to define the math model for a specific helicopter. The specific values given correspond to the AH-1S example.

```
 toujours1.dat
C>TYPE HELDATA.DAT

*****************************************************************************
* DATA FILE FOR THE AH-1S HELICOPTER PARAMETERS *
*****************************************************************************

(NAME, AIRCRAFT NAME, FS.CG, WL.CG, WT, IX, IY, IZ)
   001,    "AH-1S",  195,  75, 9000, 2593, 14320, 12330

(FS.NUB, WL.NUB, IS, OM.DAI, GAM.DM.15, R.MR, ABC.MR, RPM.MR, CDO, B.MR,C.R)
  200, 153, 0, 0, 12.5, 22, 25.65, 324, 0.012, 2, 2.25

(FS.FUS, WL.FUS, XUU.FUS, YVY.FUS, ZW.WUS)
  200, 65, -30, -275, -41

(FS.WN, WL.WN, ZUU.WN, ZW.WN, ZMAX.WN)
  200, 65, -39, -161, -65

(FS.HT, SL.HT, ZUU.HT, ZW.HT, ZMAX.HT)
  400, 65, 0, -60, -32

(FS.VT, WL.VT, YUU.VT, YVY.VT, YMAX.VT)
  490, 80, 0, -62, -50

(FS.TR, WL.TR, R.TR, ABC.TR, RPM.TR, B.TR, C.TR)
  521.5, 119, 4.25, 9.5, 1661, 2, 0.96

C>
```
C-1
The following data set is recommended for assembling information describing several flight conditions. This can be used to construct plots or tables of trim conditions and dynamic characteristics.
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